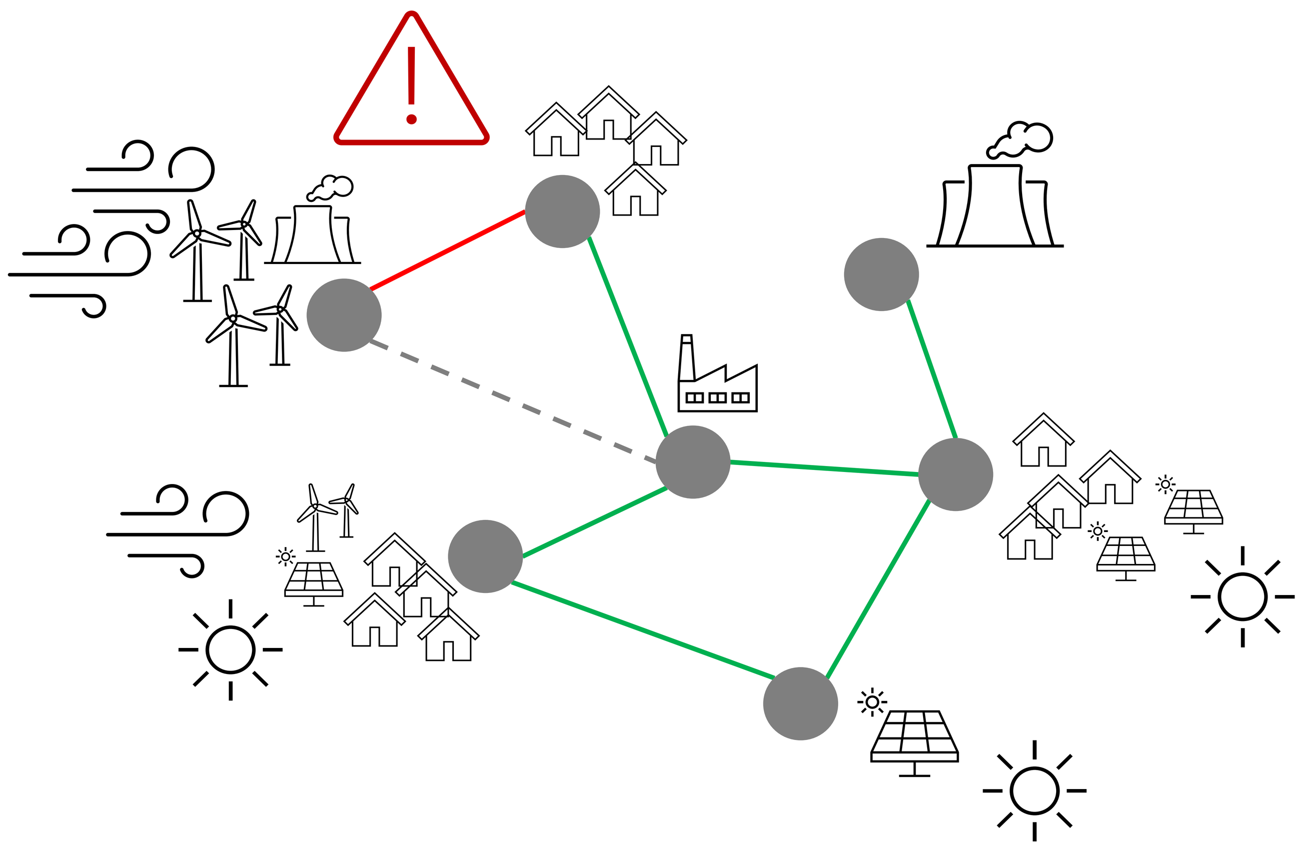
Gaussian Models for Improved Short-Term Congestion Forecasting in Power Grids

Context

As renewable energy plays a larger role in the power mix, grid congestion has become increasingly difficult to predict. While experts currently manage these disruptions, transmission system operators anticipate their growing severity and frequency, underscoring the need for advanced forecasting tools. Traditional best-estimate models associated with probabilistic power flow, however, neglect the conditional uncertainties tied to variable renewable generation, and grid topology.



An illustrative example is given with a fictitious electricity transportation network, with 7 nodes and 8 power lines. Dashed lines are disconnected, and the red power line is at risk of congestion due to wind generation on the west.

To address this, a computationally efficient Gaussian-based probabilistic layer is introduced. It can enhance a given deterministic short-term congestion forecasting model, by incorporating current grid injections, while accounting for spatial correlations among power sources. The model's agnostic design ensures broad applicability.

Method 1 : conditional gaussian + DC

Let us consider a power network with n nodes and m lines. Let $\boldsymbol{X}(t) \in \mathbb{R}^n$ be the vector of nodal injections at time t and $\widehat{\boldsymbol{X}}(t)$ a deterministic forecast of $\boldsymbol{X}(t)$ generated for time t. The objective is to model the conditional distribution $\left(\boldsymbol{X}(t+h) \mid \widehat{\boldsymbol{X}}(t+h), \boldsymbol{X}(t)\right)$. This can be written as the classical forecasting problem:

$$(\boldsymbol{Y} \mid \boldsymbol{X} = \boldsymbol{x})$$

Let $\boldsymbol{Z} = (\boldsymbol{Y},\ \boldsymbol{X}) \in \mathbb{R}^{3n}$ be the concatenated vector and $\boldsymbol{Z} \sim \mathcal{N}_{3n}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. It holds:

$$\begin{cases} (\boldsymbol{Y} \mid \boldsymbol{X} = \boldsymbol{x}) \sim \mathcal{N}_n(\boldsymbol{\mu}', \boldsymbol{\Sigma}') \\ \boldsymbol{\mu}' = \boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}(\boldsymbol{x} - \boldsymbol{\mu}_2) \\ \boldsymbol{\Sigma}' = \boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21} \end{cases}$$

Where $\boldsymbol{x}$ is the latest observation and forecast of the power system. This defines a probabilistic model for nodal injections that incorporates current grid conditions as well as spatial correlations among injections.

Under the Direct Current (DC) approximation, line flows $\mathbf{L} \in \mathbb{R}^m$ are linearly related to nodal injections: $\mathbf{L} = A\boldsymbol{X}$, where A is known as the Power Transfer Distribution Factor (PTDF) sensitivity matrix encoding network topology and line parameters. Since the conditional law is Gaussian, it holds:

$$(\boldsymbol{L} \mid \boldsymbol{X} = \boldsymbol{x}) \sim \mathcal{N}_m(A\boldsymbol{\mu}', A\boldsymbol{\Sigma}'A^T)$$

This model enables the direct computation of probabilities for line specific congestion events, such as $\mathbb{P}(\boldsymbol{L}^i > \boldsymbol{L}^i_{max})$, providing an interpretable metric for operational decision making.

Method 2 : Nataf copula + MCS

The assumption $\boldsymbol{Z} \sim \mathcal{N}_{3n}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ does not hold for some substations, especially where solar and wind productions which often exhibit bi-modal distributions. To model more precisely the marginal laws, we use a Nataf transformation paired with a Gaussian copula.

Let $\boldsymbol{W} = (\boldsymbol{X},\ \boldsymbol{Y}) \in \mathbb{R}^{3n}$, where the marginal laws of $\boldsymbol{W}$ are estimated with a Kernel Density Estimator (KDE) using N data points:

$$\forall j \in [1\ ;\ 3n], \qquad \widehat{f}_j(w) = \frac{1}{N h_j} \sum_{i=1}^{N} K\left(\frac{w - W_j(t_i)}{h_j}\right)$$

With h_j the bandwidth estimated with Silverman's method, and $K(u)$ the gaussian kernel. It is possible to transform the original variable to uniform variable using the estimated cumulative distribution functions:

$$\forall j \in [1\ ;\ 3n], \qquad U_j = \widehat{F}_j(W_j) \sim \mathcal{U}[0,1]$$

Finally, the Nataf transform is applied $Z_j = \phi^{-1}(U_j)$ to obtain normal laws. The Gaussian copula is finally:

$$(Z_1, \dots, Z_{3n}) \sim \mathcal{N}_{3n}(\boldsymbol{0}, \boldsymbol{R})$$

Once the Gaussian copula and its $\boldsymbol{R}$ parameter has been obtained, the conditional law can be derived in the same way as in method 1.

The loadflow problem is then solved with a Monte-Carlo Simulation (MCS) approximation. For samples S samples $(\boldsymbol{z}_1, \dots, \boldsymbol{z}_S) \sim \mathcal{N}_n(\boldsymbol{\mu}_{\boldsymbol{Y}|\boldsymbol{X}}, \boldsymbol{\Sigma}_{\boldsymbol{Y}|\boldsymbol{X}})$ drawn from the conditional Gaussian copula distribution, injections $\boldsymbol{x}_k = \widehat{F}^{-1}(\phi(\boldsymbol{z}_k))$ are obtained, as well as corresponding line loads $\boldsymbol{L}_k = f(\boldsymbol{x}_k)$, where f is the loadflow function.

Congestion probability are then estimated by:

$$\mathbb{P}(\boldsymbol{L}^i > \boldsymbol{L}^i_{max}) = \frac{1}{S} \sum_{s=1}^{S} 1[\boldsymbol{L}^i_s > \boldsymbol{L}^i_{max}]$$

Results

The methodology is tested on two years of real data from a section of the French power network with 459 substations and 1265 power lines. The forecasting horizon is set to 1h. RTE's forecasting model is used for $\widehat{\boldsymbol{X}}$.

The nodal injection forecast error was reduced by 25% on average on all the substations compared to the historical model. The proposed conditional Gaussian model was the best model in 99.13% of the simulations. The copula model was the best model for some simulations in May.

On the line forecasting task, the results are coherent overall, but the MCS shows better performances when abrupt changes in line loads are observed. However, proper metrics are needed for a more in-depth comparison the methods. Finally, it takes 400 s to simulate the full MCS for a single time step, compared to 1.5 s with the DC approximation. Hence, the DC approximation could serve as a cheap model to detect potentially dangerous contexts, that could be analyzed more in-depth with the MCS pipeline.

Future works

The proposed model shows promising results with injection forecasts. Future work will be dedicated to a better evaluation on power lines, to assess their relative performances. The Gaussian copula model, although very close in performance to the conditional Gaussian model, shows lacking performance, which is surprising and needs to be investigated.

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