Incentive-Based Demand-Side Flexibility in Electric Car-Sharing Systems, via Multi-Agent Multi-Level Game Optimization

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1.) Introduction

Electric car-sharing services provide short-term vehicle rentals, offering a sustainable alternative in urban mobility.

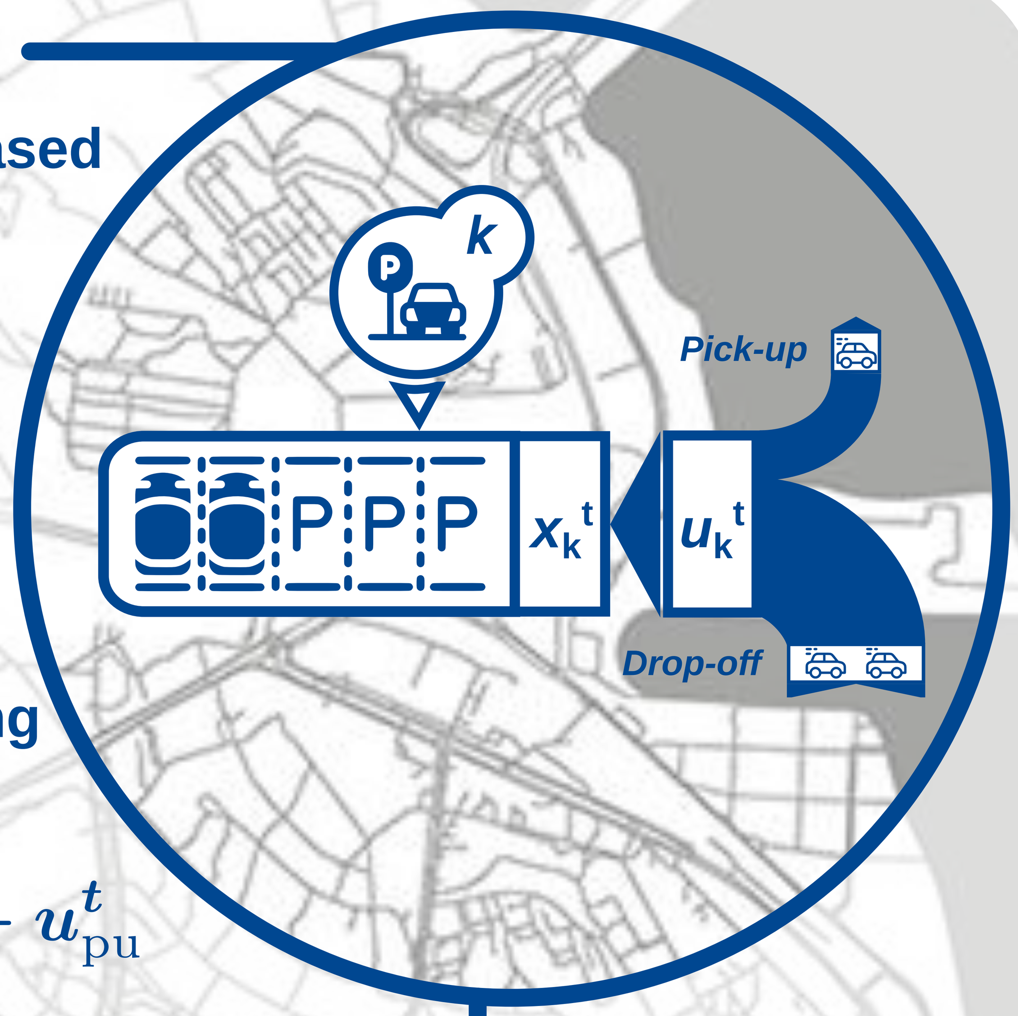
However, uneven user demand can result in unbalanced vehicle distribution, leading to unmet future demand and uneven load on the power grid during charging.

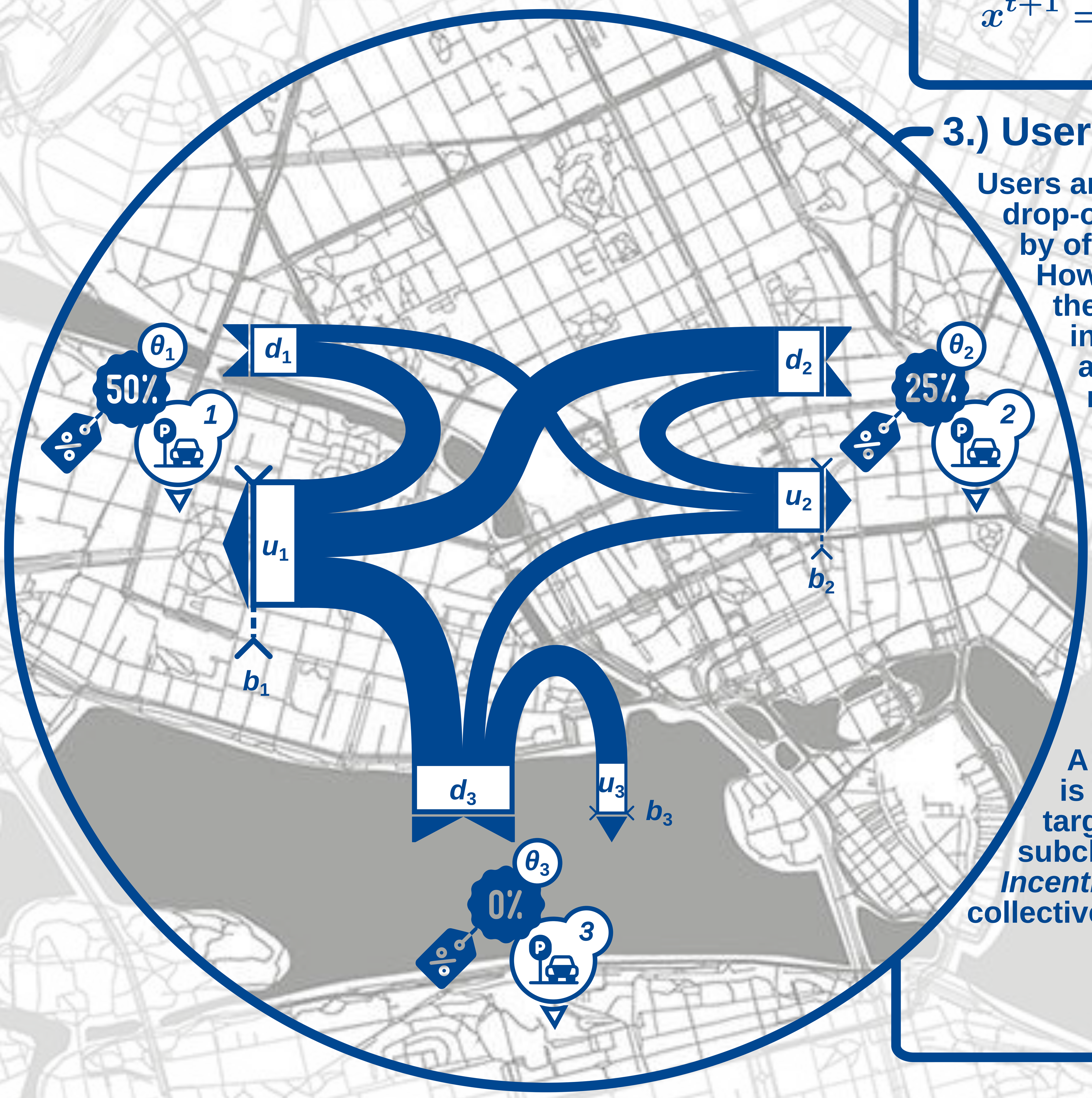
We leverage dynamic tariffs — i.e., price incentives — to rebalance vehicle distribution through demand-side control.

2.) System model

We consider a station-based car-sharing system with fixed locations offering high parking capacity and vehicle charging. Users can drop off (inflow) or pick up (outflow) cars at any station without prior reservation. These user flows satisfy the following balance equation:

$$x^{t+1} = x^t + u^t_{\text{do}} - u^t_{\text{pu}}$$





3.) User incentive response

Users are assumed to be flexible in their pick-up or drop-off locations, hence their demand can be shifted by offering price incentives — i.e. fare discounts [1]. However, vehicle availability at each station limits the redirectable user flow, affecting the overall incentive response. We model this behavior using a constrained extension of the multinomial logit model [2] in a multi-agent setting, resulting in the following generalized game:

$$(j) \begin{cases} \max\limits_{y_j \in \Delta} & y_j^\top \cdot r_j(\theta) - y_j^\top \cdot \log(y_j) \\ \text{s.t.} & \overbrace{\sum_{\forall k} d_k \cdot y_k}^{u_k} \leq b(x) \end{cases}$$

A natural solution concept for this class of games is Generalized Nash Equilibrium (GNE). Here, we target the variational GNE (v-GNE), a unique subclass of GNEs [3]. This allows us to define the *Incentive Response Map*, capturing how users collectively react to price incentives:

$$u = \text{NE}(x, \theta)$$

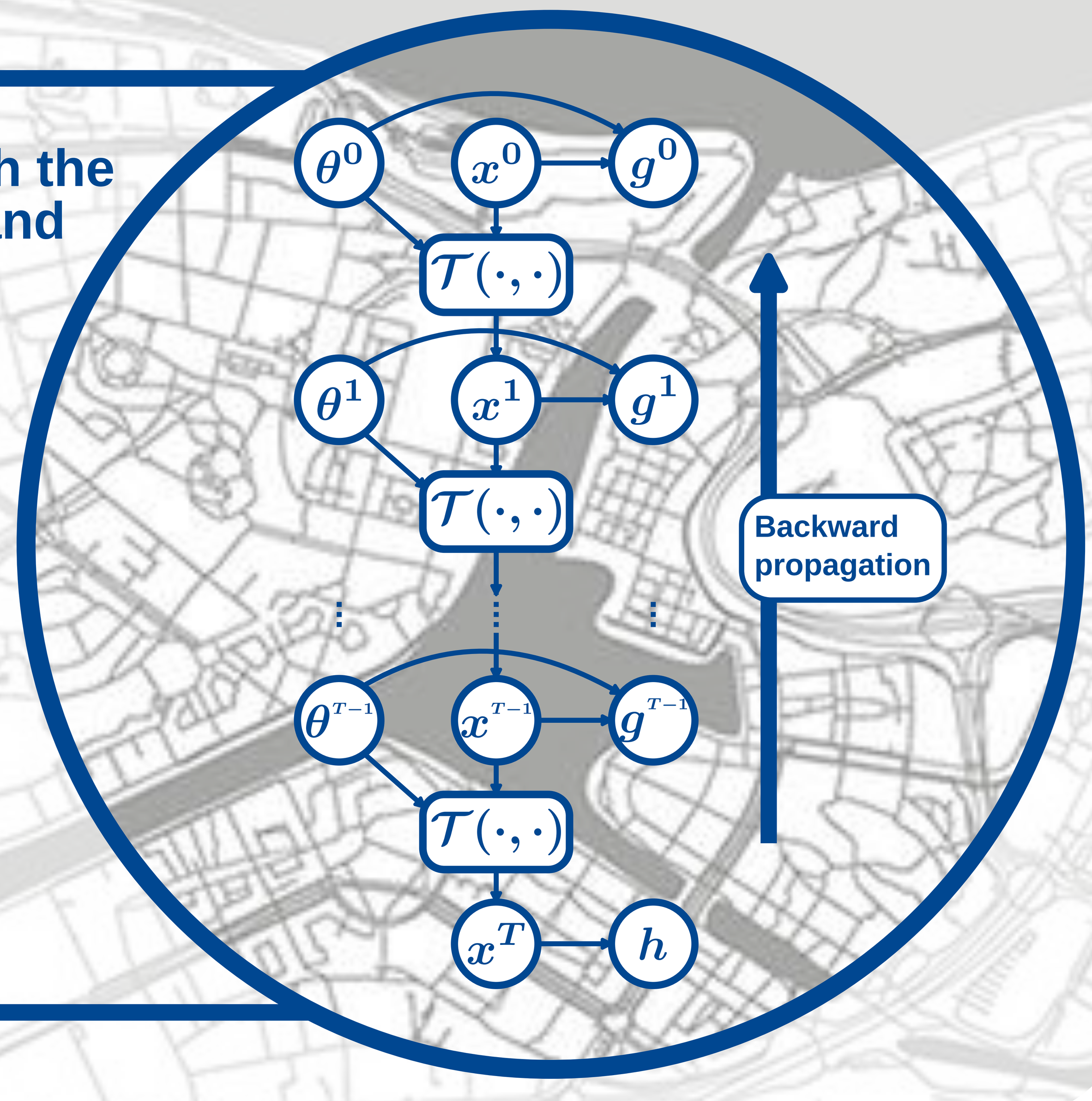
4.) Incentivized demand-side control

Using our *Incentive Response Map*, we design a demand-side flexibility signal to steer the system toward a balanced vehicle distribution — meeting future user demand and smoothing grid load from electriv vehicle (EV) charging.

This yields a Stackelberg game, with the car-sharing operator as the leader and users as followers. To solve this challenging bilevel problem, we combine differentiation through optimization with backpropagation for gradient-based optimization [4].

$$\min_{\Theta \in \Omega} \sum_{t=0}^{T-1} g^t(x^t, \theta^t) + h(x^T) \quad \text{Car-sharing operator — Leader}$$

$$\begin{aligned} \text{s.t.} \quad & x^{t+1} = A \cdot x^t + B \cdot u^t \\ & u^t = \text{NE}(x^t, \theta^t) \end{aligned} \quad \text{Users — Followers}$$



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[1] Zou, Ye, Q. Yu, D. Jiang, Y. Deng, *"Relocation Optimization for Shared Electric Vehicles: A Literature Review."*
[2] Atarashi, Kyohei, S. Oyama, H. Arai, H. Kashima, *"Box-Constrained Softmax Function and Its Application for Post-Hoc Calibration."*
[3] F. Facchinei, C. Kanzow, *"Generalized Nash Equilibrium Problems."*
[4] R. Zuliani et al., *"BP-MPC: Optimizing the Closed-Loop Performance of MPC using BackPropagation."*