Stable and Fair Uniform Price Allocations of Community Aggregation Gains in Retail Electricity Markets

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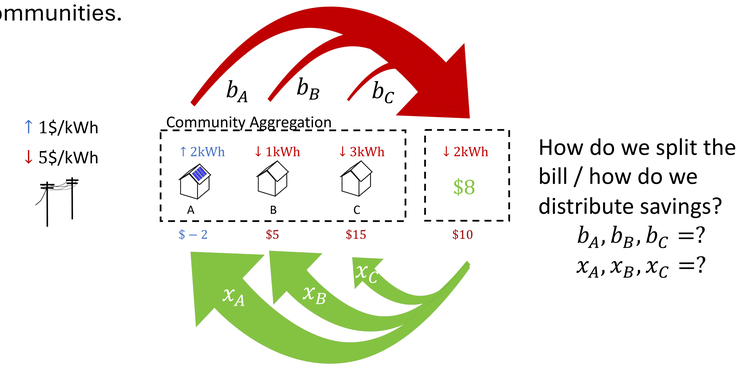
We consider an aggregation scheme where prosumers form a community that trades with the utility as a single entity. Building on cooperative game theory models from the literature, we define the uniform price core, a class of desirable distribution of savings in this context, prove that it is not empty, and find a computationally efficient imputation therein. We then compare imputations through the lens of fairness. To overcome a challenge of scalability, we apply an existing sampling methodology to optimization problems devised to obtain a fair imputation. We present theoretical results and numerical experiments.

INTRODUCTION

Setting

We consider a utility that offers different prices for buying and selling electricity. Aggregation is allowed, so there is an incentive for forming communities.



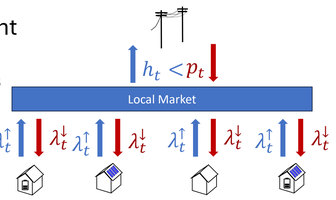
How do we split the bill / how do we distribute savings?
b_A, b_B, b_C =?
x_A, x_B, x_C =?

We want to find a **stable** cost sharing mechanism, i.e. guarantee that no subset of participants has an incentive to leave. Using a cooperative game theory model, we want an allocation in the **core of the game.**

Challenge: In general, characterizing the core requires an exponential number of constraints, as in the model we adopt as starting point [1].

UNIFORM PRICE IMPUTATIONS

The share of the bill allocated to each agent must be the result of transactions at local prices that are common to all participants



$\lambda^\uparrow$: local production price
$\lambda^\downarrow$: local consumption price

$$b_i = -\lambda^\uparrow \times \uparrow_i + \lambda^\downarrow \times \downarrow_i$$

Research questions

- Can we guarantee that stable uniform prices $\lambda^\uparrow$, $\lambda^\downarrow$ always exist? i.e. is the uniform price core non-empty?
- Can we find them efficiently?

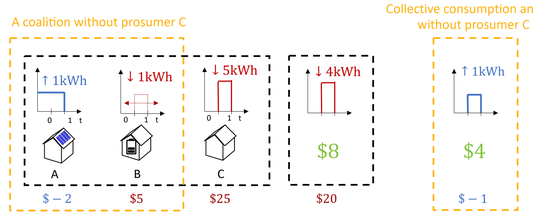
Results

- We prove that solving the LP of the aggregation coordination problem gives an allocation in the uniform price core when pricing electricity at the shadow price of the power balance constraint.
- We call this imputation the **shadow price imputation**.

FAIRNESS IN COST SHARING

Example

↑ 1$/kWh
↓ 5$/kWh



If we make $\lambda_t^\uparrow = \lambda_t^\downarrow = \lambda_t$, $(b_A, b_B, b_C) = (-5 - \lambda_0, \lambda_0, 25)$
Customer C always gets $0 savings
But without C, there would only be $4 of collective savings

If we make $\lambda_0^\uparrow = \lambda_1^\uparrow = \$3$, $\lambda_0^\downarrow = \$3, \lambda_1^\downarrow = \$4.6$, $(x_A, x_B, x_C) = (4,2,2)$
Everyone gets something!

The shadow price imputation can be unfair. Allowing different selling and buying prices can help recover fairness. We formalize that with optimization.

An optimization framework for fairness

Introduce a function *f* that represents preference over imputations and solve:

$$\min f(b, \lambda^\uparrow, \lambda^\downarrow)$$
$$s.t.$$
$$\sum_{i \in S} b_i \leq C(S), \forall S \subseteq N$$ Stability (definition of the core)
⚠ Exponential number of constraints ⚠
$$b_i = -\lambda^\uparrow \times \uparrow_i + \lambda^\downarrow \times \downarrow_i, i \in N$$ Uniform price

Research questions

- Can we efficiently find an optimal stable uniform price allocation?

Results

- We don't have an efficient way of solving this problem, but we do if we relax a little the stability requirement

A statistical learning approach

Consider only a sample of all possible subsets: can we get a "good enough" imputation with a "small enough" number of samples?

$$\forall S \subseteq N. \quad \forall S \in \Sigma$$

A probabilistic relaxation of the core:

An imputation where participants can find alternatives which are better than the proposed allocation by more than ϵ with probability $< \delta$

Formally:

(ϵ, δ)-probably, approximately stable core: an imputation *b* for which:

$$\mathbb{P}_{S \sim \mathcal{S}}\left((1-\epsilon) \sum_{i \in S} b_i \leq C(S) \right) \geq 1 - \delta$$

Drawing from [3], we prove we can find such an imputation with $\mathcal{O}(\log(N))$* samples.

*omitting terms with ϵ, δ

NUMERICAL RESULTS

We create testcases with 4 to 50 prosumers with 1 year of hourly data from [4].

We compare different fairness functions for N=4,8. Metrics improve w.r.t. shadow imputation.

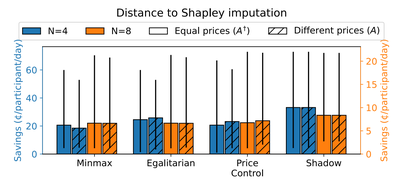


Figure 1. Comparison of four mechanisms for 365 days. Main bars: avg. Error bars: min-max range

We test the sampling methodology with up to N=50 participants.

A few hundred samples suffice to obtain imputations in the approximately stable core for all cases.

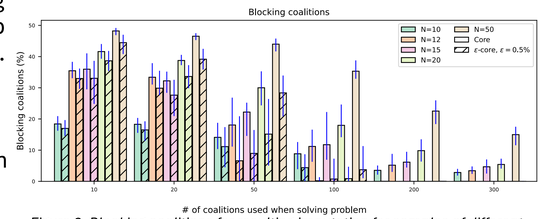


Figure 2. Blocking coalitions for resulting imputation for samples of different size. 10 repetitions. Main bars: avg. Error bars: min-max range

CONCLUSIONS

- We formalize the **uniform price core** and prove it is not empty
- We propose a computationally scalable methodology to obtain the **shadow price imputation**, an allocation in the uniform price core
- We propose an optimization methodology for finding fair allocations and overcome its scalability challenge by proposing a sampling methodology from the statistical learning literature.
- More in the paper [5]: we prove this game is a linear production game in the sense of [2] and the shadow price allocation is a *dual* or *Owen* allocation; we discuss the stochastic case and the utility's response.

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[3] T. Yan, A. Procaccia, "If You Like Shapley Then You'll Love the Core", AAAI Conference on Artificial Intelligence, (2021)
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