Physics-informed neural network-based reduced order electric water heater modeling for demand response

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Background

- The aggregated flexibility of electric water heaters (EWHs) has the potential to provide significant benefits to the grid through local flexibility or reserve markets.
- An effective control strategy for maximizing EWH flexibility requires a highly accurate, computationally inexpensive model.
- Various methods exist for modeling an EWH to support demand response applications[1,2], each with unique characteristics and trade-offs:

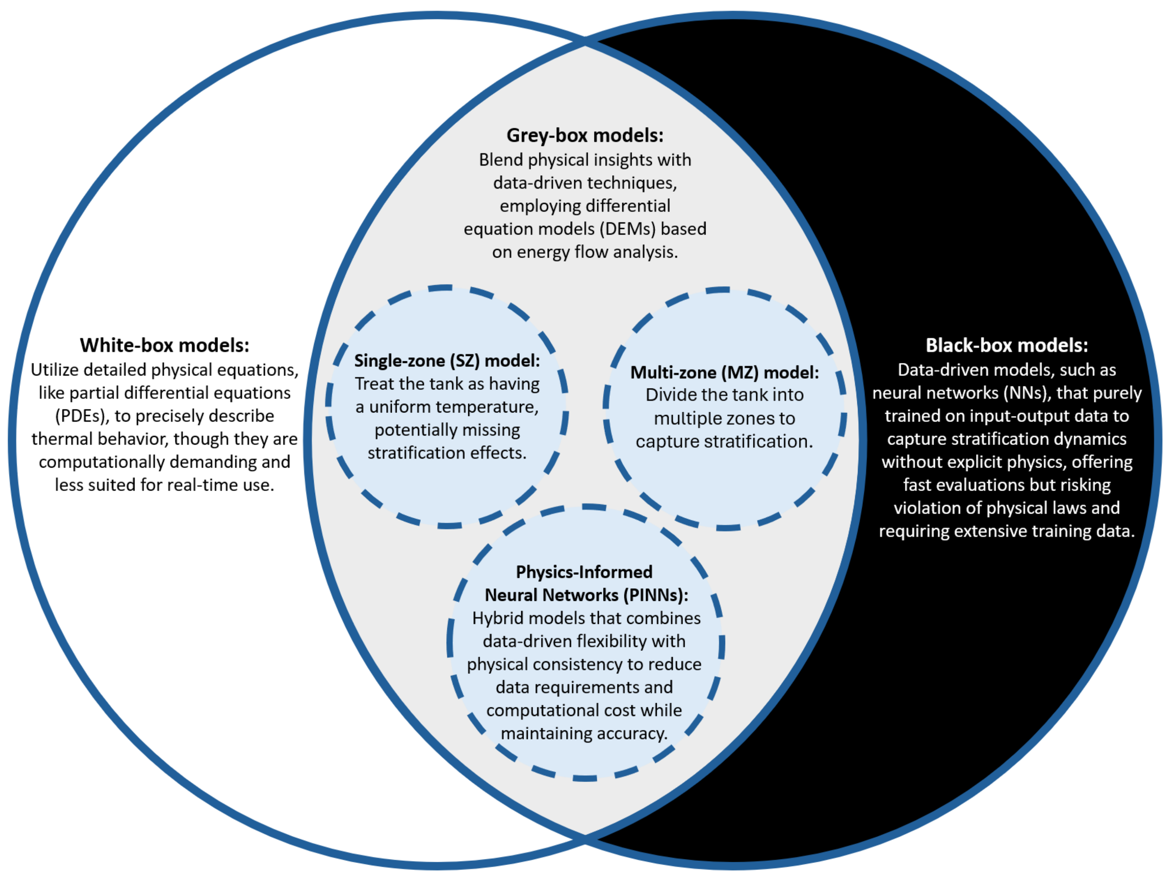

Fig. 1: Comparison of Modeling Approaches for Electric Water Heaters.

Methodology

- Multi-zone (MZ) model discretizes EWH into two or more thermal zones. The rate of change of water temperature along the EWH tank with N thermal zones is described using a system of N ordinary differential equations (ODEs), given as[3] :

$$\dot{T}_i(t) = \frac{1}{C}\Big[Q(t) - GA_s(T_i(t) - T_{amb}(t)) - \rho c_p W_d(t)(T_i(t) - T_{i-1}(t)) + \frac{kA_c(T_{i+1}(t) - T_i(t))}{z_{i+1} - z_i} - \frac{kA_c(T_i(t) - T_{i-1}(t))}{z_i - z_{i-1}}\Big]$$

$$For \quad i = 1, 2..., N$$

$$Where \quad k = \begin{cases} k_z & T_{i+1}(t) \le T_i \\ 0 & T_{i+1}(t) > T_i \end{cases}$$

$\dot{T}_i(t)$	Rate of change of temperature of zone i [°C/s]
A_c	Cross sectional area of the tank [m^2]
A_s	Surface area of the tank [m^2]
C	Thermal capacity [W s/°C]
G	Thermal conductance [W/m^2 °C]
k	Thermal conductivity of the water [W/m°C]
$Q(t)$	Power consumption [W]
t	Time step [s]
$T_i(t)$	Average temperature of zone i at time t [°C]
T_{amb}	Ambient temperature [°C]
T_{in}	Inlet water temperature [°C]
$W_d(t)$	Hot water demand at time t [kg/s]
z_i	Height of the ith zone center [m]

- The high dimensionality of MZ DEM or physics-informed neural network (PINN)-based model with 12 thermal zones leads to the problem being computationally expensive.
- To address this, a lower-dimensional EWH model that retains necessary details is required. Simulating a subset of thermal zones—including the first and last zones spanning the tank height—is sufficient to generate optimal control actions.
- The PINN-ROM (reduced order model) approximates the dynamics of the selected thermal zones. Systematic testing of different input features and architectures highlighted the need for a custom architecture with tailored inputs.
- Each zone's NN outputs the temperature of its corresponding zone without resolving temperature inversion (TI). A custom NN is introduced specifically to handle TI.
- A recursive training approach is used, where the model output is fed back as input for the next sub-period.

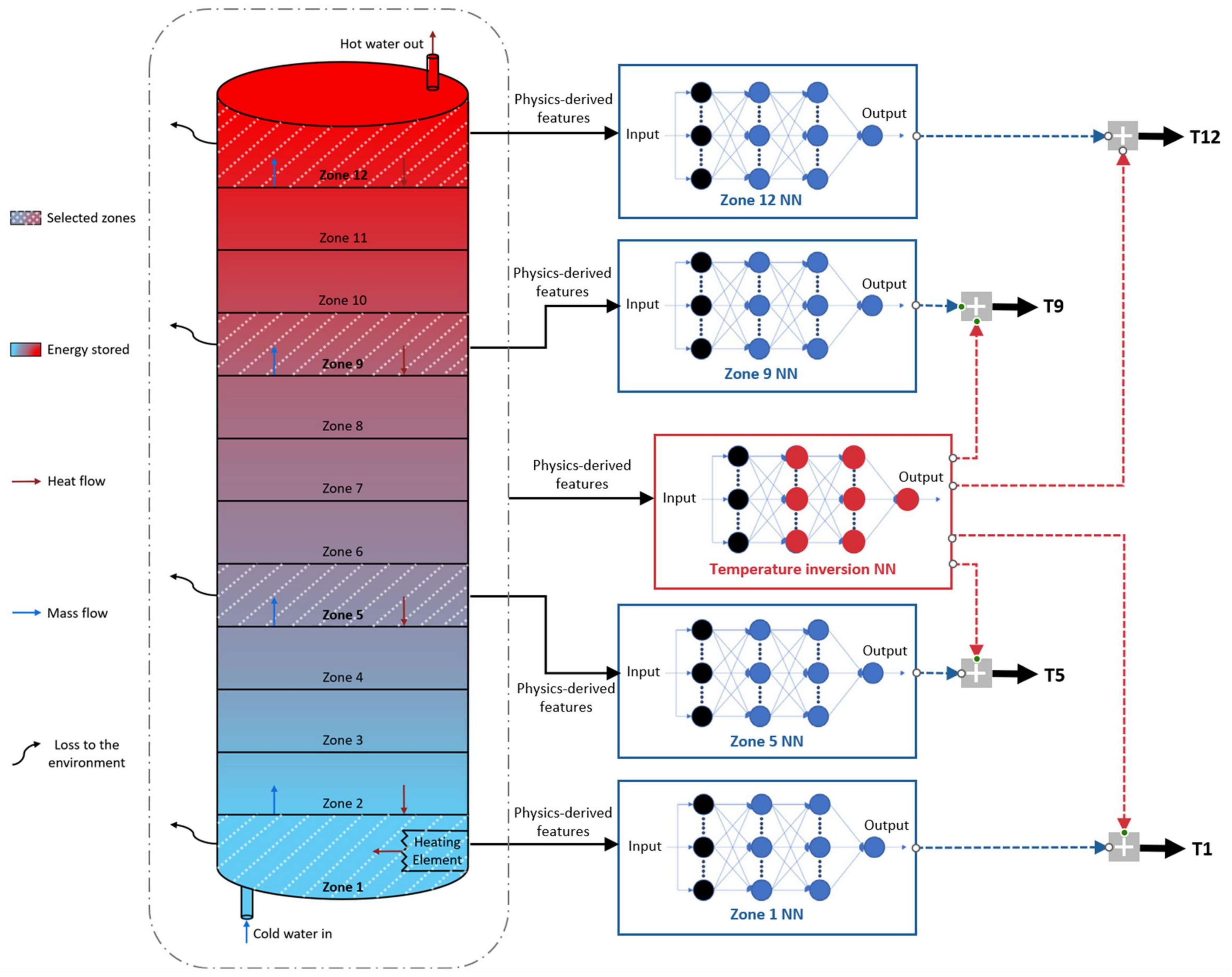

Fig. 2: Schematic representation of energy and mass flow in MZ model and proposed architecture for PINN-ROM and the system of NNs. Not all neurons and connections are shown for figure legibility.

Research Objectives

- Designing an appropriate architecture, training data, and methodology to develop a model suitable for aggregated demand response (DR) applications.
- Evaluating performance in terms of accuracy and computational efficiency compared to traditional physics-based model.

Preliminary Results

- The performance of PINN-ROM model, trained to represent solutions of the ODEs in MZ differential equation model (DEM), was compared with the standard Runge-Kutta (RK4) numerical solver for ODEs.
- To evaluate accuracy, the average Root Mean Squared Error (RMSE) was calculated for each zone across all experiments, and the results for the 3kW EWH are presented in Table 1.

Table 1: Average RMSE values of PINN-ROM compared with the MZ DEM for all days in testing

RMSE (°C)	Zone 1	Zone 5	Zone 9	Zone 12	Overall
PINN-ROM	1,06	2,05	1,30	1,39	1,45

- The developed models exhibit sufficiently low error for effective utilization in DR control, making them viable alternatives to MZ DEM.

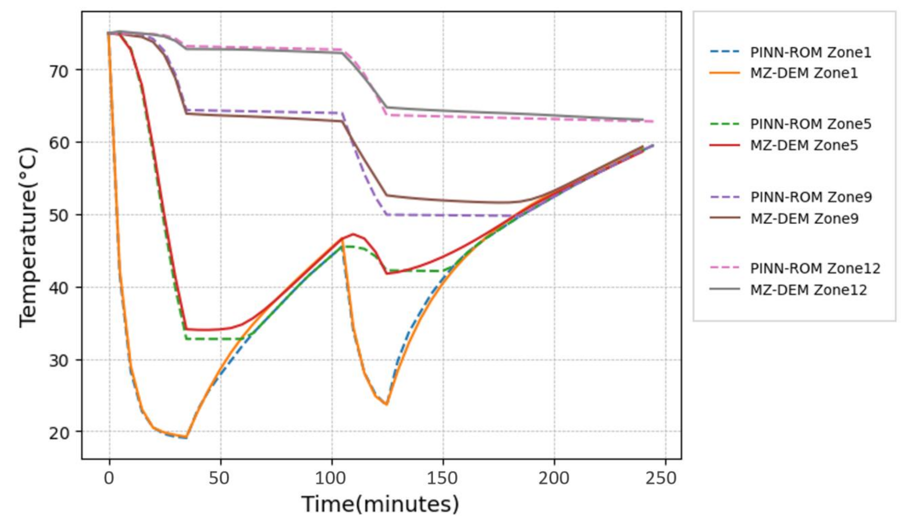

Fig. 3: Comparison of water temperature predictions from PINN-ROM with simulated values from MZ-DEM during an event (representing a real-world scenario) lasting approximately 250 minutes.

- The number of Floating Point Operations (FLOPs) is theoretically calculated to evaluate the computational cost of PINN-ROM model compared to continuous MZ DEM. The results are presented in Table 2.

Table 2: FLOPs for a single prediction using the MZ DEM and PINN-ROM

Model	FLOPs
MZ DEM (RK4)	4250
PINN-ROM	1185

- PINN-ROM requires 1185 FLOPs, which is 0.72 times less than RK4 method. In other words, PIN-ROM is 3,58x faster than the RK4 method.

Conclusion & Future work

- The present work is focused on developing a PINN-based ROM with a recursive training strategy, simplified architecture, physics-derived features, and a custom NN to resolve temperature inversion.
- PINN-ROM outperforms the traditional MZ DEM in efficiency, as demonstrated by performance evaluations using simulation data designed to represent real-world conditions.
- Future work includes utilizing the immense potential of the PINN-ROM to develop efficient scheduling and control strategies for large-scale aggregation of distributed EWH flexibility and participation in day-ahead and balancing markets.

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