Conic Programming Formulation for Optimal Hybrid Sizing in AC Grids

Alan Ortiz Contreras, Mohamed Badaoui, David Sebastián Baltazar

Instituto Politécnico Nacional
Escuela Superior de Ingeniería Mecánica y Eléctrica Unidad Zacatenco
Sección de Estudios de Posgrado e Investigación (SEPI – ESIME Zac)
Av. Luis Enrique Erro S/N – C.P. 07738 – Mexico City
aortizc0903@alumno.ipn.mx, mbadaoui@ipn.mx, dsebasti@ipn.mx

Abstract

This research presents a set of models for optimal sizing and selection of hybrid renewable energy systems in Electrical Power System Mexican through deterministic and convex-optimization techniques. A mixed-integer linear programming (MILP) model identifies the best combination of photovoltaic arrays, wind turbines, and battery storage for the load areas in Tamaulipas and Oaxaca. In contrast, a mixed-integer second-order cone programming (MISOCP) model—based on a modified 5-bus PJM network scaled to Mexican demand profiles—integrates time-value-of-money factors, locational market revenues, and conic relaxations of network constraints. Preliminary simulations reveal characteristic battery cycling patterns, seasonal shifts in the renewable mix, and power sales opportunities while highlighting the need for faster solution methods and enhanced voltage-stability enforcement.

Finally, the future research outlines several extensions under demand uncertainty—scenario-based stochastic programming, chance constraints, two-stage recourse models, CVaR-based risk aversion, and neural-network demand forecasting—designed to ensure resilient, cost-effective integration of renewables under real-world variability.

Keywords: *Second-order conic relaxation, Optimal sizing, Hybrid System, Photovoltaic, Wind, energy storage systems, AC optimal Power flow.*

1. Previous Work: The Optimal Selection of Renewable Energy Systems Based on MILP for Two Zones in Mexico

Developed *MILP model* for hybrid energy sizing:

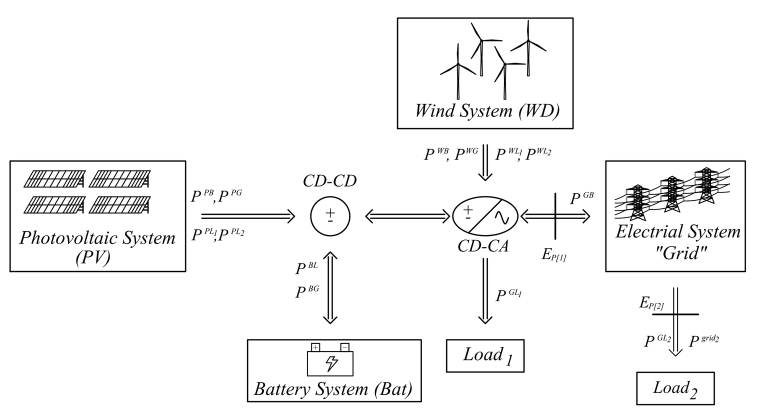


Figure 1: *Configuration of a photovoltaic wind battery generation system, considering two loads.*

Objective Function

maximize

$$N_{PV} := \sum_{h\in H} \frac{1}{(1+r)^h} \left[\sum_{p\in P} \underbrace{\left(\text{Energy sales revenue}\right)}_{\text{Assets}} - \underbrace{\left(\text{System costs and Energy purchases}\right)}_{\text{Liabilities}} \right] \quad (1)$$

Assets:

- PV to grid:
- Wd to grid:
- Bat to grid:
- Loads revenue:

Liabilities:

- CAPEX:
- FOM:
- Grid purchases:
- Bat charging:
- Grid usage fee:

All terms discounted at rate r over planning horizon H

Key Constraints

System Selection: $\Longrightarrow$

$$\sum_{i\in I} y_i \le N^{PV}, \quad \sum_{j\in J} w_j \le N^{Bat}, \quad \sum_{k\in K} z_k \le N^{Wd} \qquad \forall p\in P \quad (2)$$

PV Generation: $\Longrightarrow$

$$y_i \cdot A_i \cdot GHI_p \cdot \eta_{ip}^{PV} = P_{ip}^{PG} + P_{ip}^{PL_1} + \sum_{j\in J} P_{ijp}^{PB} + P_{ip}^{PL_2} \qquad \forall p\in P \quad (3)$$

WD Generation: $\Longrightarrow$

$$z_k \cdot G_k^{Ag} \cdot N_{Ag} = P_{kp}^{WG} + P_{kp}^{WL_1} + \sum_{j\in J} P_{kjp}^{WB} + P_{kp}^{WL_2} \qquad \forall p\in P \quad (4)$$

Power Balance: $\Longrightarrow$

$$P_p^{GL_1} + \sum_{i\in I} \eta_i^{PVin} P_{ip}^{PL_1} + \sum_{j\in J} P_{jp}^{BL_1} + \sum_{k\in K} P_{kp}^{WL_1} = P_p^{Load_1} \qquad \forall p\in P \quad (5)$$

$$\Longrightarrow \quad \sum_{i\in I} \eta_i^{PVin} P_{ip}^{PL_2} + \sum_{k\in K} P_{kp}^{WL_2} + P_p^{GL_2} = P_p^{Load_2} \qquad \forall p\in P \quad (6)$$

No Simultaneous Charge/Discharge:

$$P_{ijp}^{PB} + P_{kjp}^{WB} + P_{jp}^{GB} \le M y_{jp}^B \quad (7)$$

$$P_{jp}^{BG} + P_{jp}^{BL_1} \le M(1 - y_{jp}^B) \quad (8)$$

Battery Dynamics:

$$B_{jp} = (1-\beta_j) \left[\eta^{CC} \eta^{C} \left(\sum_i P_{ijp}^{PB} + \eta^{Bin} \left(P_{jp}^{GB} + \sum_k P_{kjp}^{WB} \right) \right) - \frac{P_{jp}^{BG} + P_{jp}^{BL_1}}{\eta^{Bin} \eta^{CC} \eta^{D}} \right] \qquad \forall p\in P \quad (9)$$

$$SOC_{jp} = SOC_{j(p-1)} + B_{jp} \qquad \forall p\in P \quad (10)$$

$$w_j SOC_j^{min} \le SOC_{jp} \le w_j SOC_j^{max} \qquad \forall p\in P \quad (11)$$

$$(y_{jp}^B - 1) DR_j \le B_{jp} \le y_{jp}^B CR_j \qquad \forall p\in P \quad (12)$$

No Grid Export While Supplying $Load_1$:

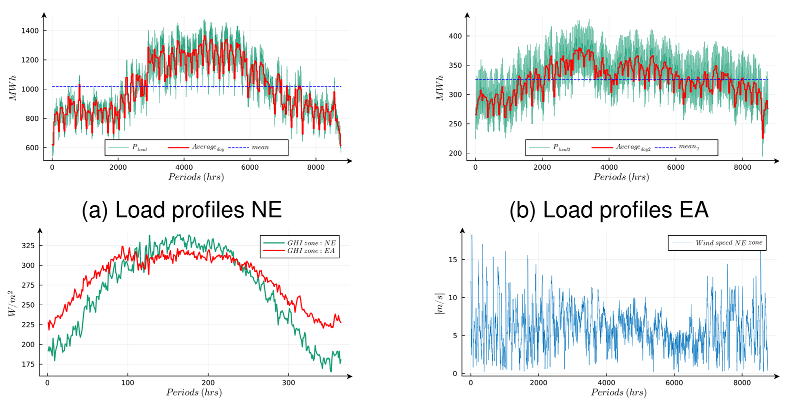
$$P_p^{GL_1} \le M y_p^G \quad (13)$$

$$\sum_i P_{ip}^{PG} + \sum_j P_{jp}^{BG} + \sum_k P_{kp}^{WG} \le M(1 - y_p^G) \quad (14)$$

Case Study & Key Results

Study Area & Inputs

- Locations
  - NE: Tamaulipas (24°N, 98°W)
  - EA: Oaxaca (16°N, 95°W)
- Resources
  - Solar GHI: 4.8 (NE), 5.2 (EA) kWh/m^2/day
  - Wind: 8.2 (NE), 9.1 (EA) m/s @100 m
- Loads
  - Zone 1: 1017 MW (15%)
  - Zone 2: 325 MW (5%)



(a) Load profiles NE
(b) Load profiles EA
(c) Global horizontal irradiation
(d) Wind speed 100 m

Figure 2: *Graphical summary: (a,b) demand profiles; (c,d) GHI and wind speed.*

Cases comparison

Case	PV	Wind	Battery	NPV (US$ B)	Savings (MW) L1/L2
1	1000	630	-	167.9	424.6, 264.6
2	1150	1030	-	131.1	128.6, 685.4
3	1200	200	1000	146.1	388.8, 208.0
4	1500	200	1000	101.1	502.1, 127.0

Table 1: *Comparative summary of the four cases*

- **Cases:**
  1. Hybrid system, load 1 in NE and load 2 in EA zone.
  2. Hybrid system, load 1 in EA and load 2 in NE zone.
  3. Hybrid system, load 1 in NE and load 2 in EA zone, forced.
  4. Hybrid system, load 1 in EA and load 2 in NE zone, forced.

- **PV** (MW)
- **Wind** (MW)
- **Bat** (MWh)
- **NPV** US$ billion (B)
- **Saving L1 / L2** (MW)

Sensitivity, Storage & Policy

- Sensitivity
  - +25% Load 2 → -18% revenue
  - -10% CAPEX → +12% NPV
  - IR: 3%→$210 B, 16% → $78 B.
- Storage
  - +1000 MWh batt. → +$35 B CAPEX
  - +22% self-consumption
  - payback→8.9 yr
- Policy Satisfies
  - 35% clean by 2024
  - +$2.3 B trade
  - -38% congestion NE
- Tech 8 MW turbines commercial
  - 45% local components
  - 22 PV parks > 500 MW.

2. Actual work: Sizing SOCP model

Developed *SOCP model* for hybrid energy sizing:

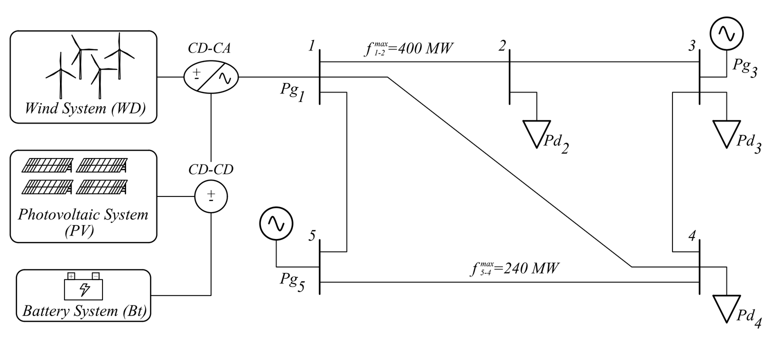


Figure 3: *Power energy 5-PJM system test.*

Objective Function

maximize

$$\begin{aligned} N_{PV} :=& -(\textit{HS investment} + \textit{Annual maintenance}) \\ &+ (\textit{Demand Profit} + \textit{Network Usage Profit}) \\ &- (\textit{Generation expenditure}) \end{aligned} \quad (15)$$

OPF constraints

$$\sum_{i\in I} \eta^{Pin} P_{i,p}^{PG} + \sum_{k\in K} P_{k,p}^{WG} + \sum_{j\in J} \left(\frac{P_{j,p}^{BG}}{\eta^{Bin} \eta^{CC} \eta^{D}} \right) = Pg_{1,p} \qquad \forall p\in P \quad (16)$$

$$Pg_{1,p} - Pd_{1,p} - \sum_{j\in J} P_{j,p}^{GB} = RC_{11}^p G_{11} + \sum_{m\in \mathscr{L}_1} (RC_{1m}^p G_{1m} - IC_{1m}^p B_{1m}) \qquad \forall p\in P \quad (17)$$

$$Pg_{n,p} - Pd_{n,p} = RC_{nn}^p G_{nn} + \sum_{m\in \mathscr{L}_n} (RC_{nm}^p G_{nm} - IC_{nm}^p B_{nm}) \qquad \forall n\in \mathscr{N} \setminus \{1\}, p\in P \quad (18)$$

Net Present Value (NPV) Components:

- **Assets:**
  - *Demand Profit*: Energy sales revenue to loads 2-4.
  - *Network Usage Profit*: Grid congestion revenue.
- **Liabilities:**
  - *HS Investment*: CAPEX for renewable systems.
  - *Annual Maintenance*: FOM costs.
  - *Generation Expenditure*: Conventional generation costs.

Key Financial Parameters:

- $f_{rH} = \frac{1-(1+r)^{-H}}{r}$: Present value factor (11% rate, 30-year horizon)
- $Base$: Power base (100 MVA conversion)
- Ie: 2.23% annual demand growth
- N^{wt}: 50 turbines per wind farm

Remark 1 *This convex formulation enables efficient optimization while capturing:*
1. Time-value of money through f_{rH}
2. Technology-specific costs (CAPEX/FOM ratios)
3. Market dynamics (LMP-based revenue streams)
4. Network constraints via conic relaxation

$$Qg_{n,p} - Qd_{n,p} = -RC_{nn}^p B_{nn} - \sum_{m\in \mathscr{L}_n} (RC_{nm}^p B_{nm} + IC_{nm}^p G_{nm}) \qquad \forall n\in \mathscr{N}, p\in P \quad (19)$$

$$\underline{Pg_n} \le Pg_{n,p} \le \overline{Pg_n}, \qquad \underline{Qg_n} \le Qg_{n,p} \le \overline{Qg_n} \qquad \forall n\in \mathscr{N} \setminus \{1\}, p\in P \quad (20)$$

$$\left(\overline{f_{nm}}\right)^2 \ge (Pf_{nm}^p)^2 + (Qf_{nm}^p)^2 \qquad \forall n\in \mathscr{N}, p\in P \quad (21)$$

$$RC_{nm}^p = RC_{mn}^p, \quad IC_{nm}^p = -IC_{mn}^p \qquad \forall (n,m)\in \mathscr{N}, p\in P \quad (22)$$

$$(RC_{nm}^p)^2 + (IC_{nm}^p)^2 \le RC_{nn}^p RC_{mm}^p \qquad \forall (n,m)\in \mathscr{N}, p\in P \quad (23)$$

$$\left(\underline{V_n}\right)^2 \le RC_{nn}^p \le \left(\overline{V_n}\right)^2 \qquad \forall n\in \mathscr{N} \setminus \{1\}, p\in P \quad (24)$$

Test System Overview:

- Modified PJM 5-bus network (MATPOWER base)
- 2 conventional generators (520-600 MW range)
- Scaled demand profiles from Mexican regions:
  - Bus 2: 3.125% NE demand (15.9 MW baseline)
  - Bus 4: 3.125% EA demand (10.2 MW baseline)

Renewable Integration Framework:

- 5 PV-storage configurations under evaluation
- 16 wind turbine models in technical review
- Preliminary CAPEX estimates:
  - PV: $1.05M/MW (subject to market analysis)
  - Storage: $0.83M/MWh (technology-dependent)
  - Wind: $1.79M/MWh (subject to market analysis)

Policy Implications:

- 450 MW offshore wind potential demonstrated
- 42% reduction in conventional generation
- Aligns with Mexico's 35% renewable energy target by 2035

Preliminary Insights

- **Current Model Status:**
  - SOCP implementation with Gurobi/JuMP
  - Current runtime: 5.7 hr (optimization needed)
- **Initial Observations:**
  - Storage cycling pattern emerging:
    - Night charging (18:00-08:00)
    - Afternoon discharge (12:00-18:00)
  - Seasonal mix variations detected:
    - Summer: PV-dominant pattern
    - Winter: Wind-PV balance shift

Next Development Steps

- Model refinement priorities:
  - Voltage stability constraints (currently 0.95-1.08 p.u.)
  - Constraints to avoid transmission line congestion
- Validation requirements:
  - Field data reconciliation for wind profiles
  - Battery degradation model integration
  - Multi-year performance projections

3. Modeling Extensions for Stochastic Sizing under Demand Uncertainty

1. **Deterministic Benchmark Model**
   The Sizing SOCP model formulation, where demand is treated as a known constant value.

$$\sum_{i\in I} \eta^{Pin} P_{i,p}^{PG} + \sum_{k\in K} P_{k,p}^{WG} + \sum_{j\in J} \left(\frac{P_{j,p}^{BG}}{\eta^{Bin} \eta^{CC} \eta^{D}} \right) = Pg_{1,p} \quad \forall p \in P \quad (25)$$

2. **Stochastic Programming with Discrete Demand Scenarios**
   The demand is represented by a finite set of discrete scenarios, each associated with a probability. We define a set of demand scenarios $\{d_p\}$ with corresponding probabilities p_s. The power balance equation is modified to include these demand scenarios:

$$\sum_{i\in I} \eta^{Pin} P_{i,p}^{PG} + \sum_{k\in K} P_{k,p}^{WG} + \sum_{j\in J} \left(\frac{P_{j,p}^{BG}}{\eta^{Bin} \eta^{CC} \eta^{D}} \right) = Pg_{1,p} + d_p \quad \forall p \in P, \forall s \in S \quad (26)$$

3. **Chance-Constrained Sizing SOCP model**
   For chance-constrained programming, we introduce a probability level α to represent the desired confidence level that the power balance will hold for all demand scenarios. The power balance constraint becomes:

$$\mathbb{P}\left(\sum_{i\in I} \eta^{Pin} P_{i,p}^{PG} + \sum_{k\in K} P_{k,p}^{WG} + \sum_{j\in J} \left(\frac{P_{j,p}^{BG}}{\eta^{Bin} \eta^{CC} \eta^{D}} \right) - d_p \ge Pg_{1,p} \right) \ge \alpha \quad \forall p \in P \quad (27)$$

4. **Robust Sizing SOCP model with Demand Uncertainty Sets**
   Robust optimization ensures feasibility for all realizations of demand within a predefined uncertainty set $\mathcal{D}$, where demand is constrained as follows:

$$d_p \in \mathcal{D} \quad \forall p \in P \quad (28)$$

   Thus, the constraint must hold for all possible realizations of demand in the set $\mathcal{D}$.

5. **Risk-Averse Stochastic Sizing via Conditional Value-at-Risk (CVaR)**
   In this approach, we minimize the expected cost while penalizing extreme demand realizations. The objective function incorporating CVaR is given by:

$$\min \mathbb{E}[\text{Cost}] + \lambda\, \text{CVaR}_\alpha(d_p) \quad (29)$$

   where λ is a risk-aversion parameter and $\text{CVaR}_\alpha(d_p)$ is the conditional value-at-risk at level α.

DTU PES Summer School
Future Energy Systems: Leveraging Advanced Optimization, Data Sharing and AI for Market Design and Operation
May 18 – 23 2025