

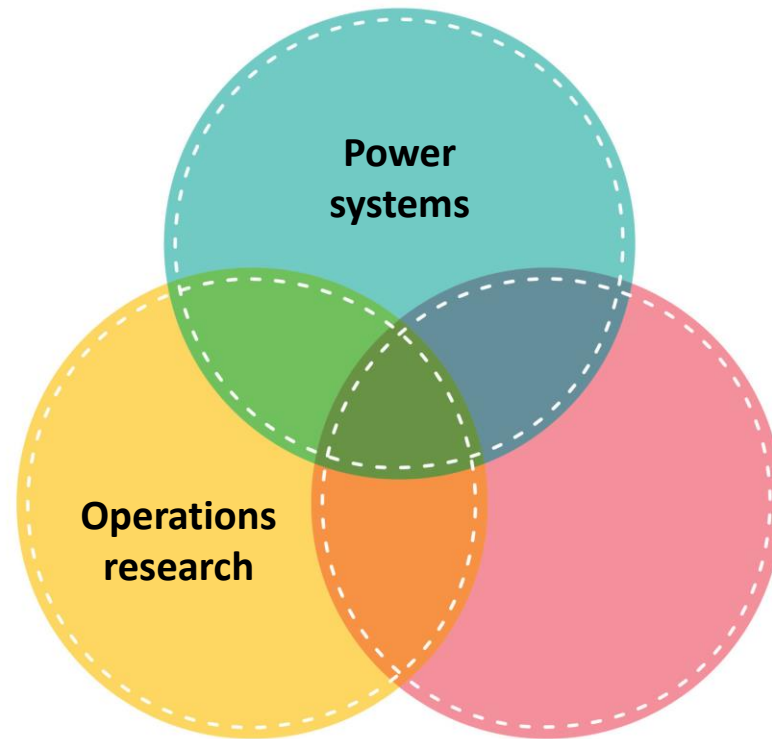
DTU PES Summer School 2024

How to Optimally Bid in Nordic Ancillary Service Markets?

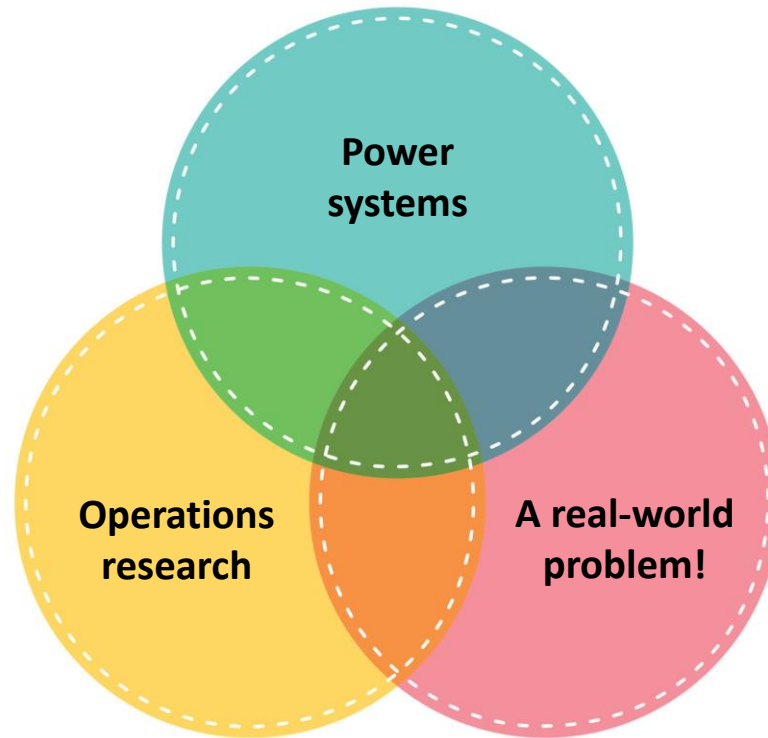
Jalal Kazempour

May 30, 2024

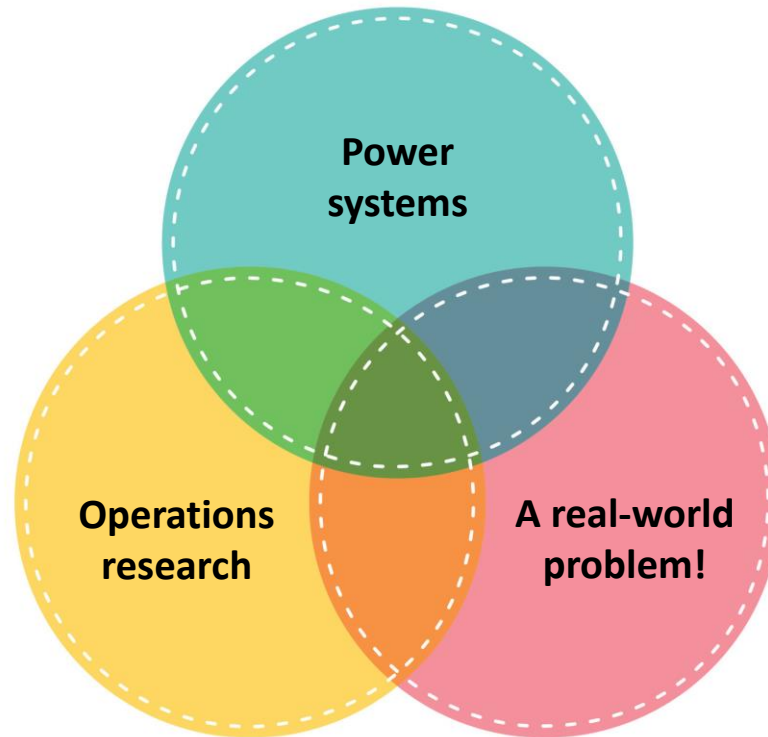
$$f(x+\Delta x) = \sum_{i=0}^{\infty} \frac{(\Delta x)^i}{i!} f^{(i)}(x)$$
$$\int_a^b \varepsilon \Theta + \Omega \int \delta e^{i\pi} = \{2.7182818284\}$$
$$\sqrt{17}$$
$$\infty$$
$$\chi^2$$
$$\Sigma$$
$$!$$



Can they live together?



Can they live together?



Hope this lecture will inspire you to think how to use (advanced) operations research for **real-world problems** in power systems/markets.

Credit of this work goes to:



Peter Gade
(Industrial PhD student
with IBM and DTU)



Gustav Lunde
(former MSc student)



Emil Damm
(former MSc student)

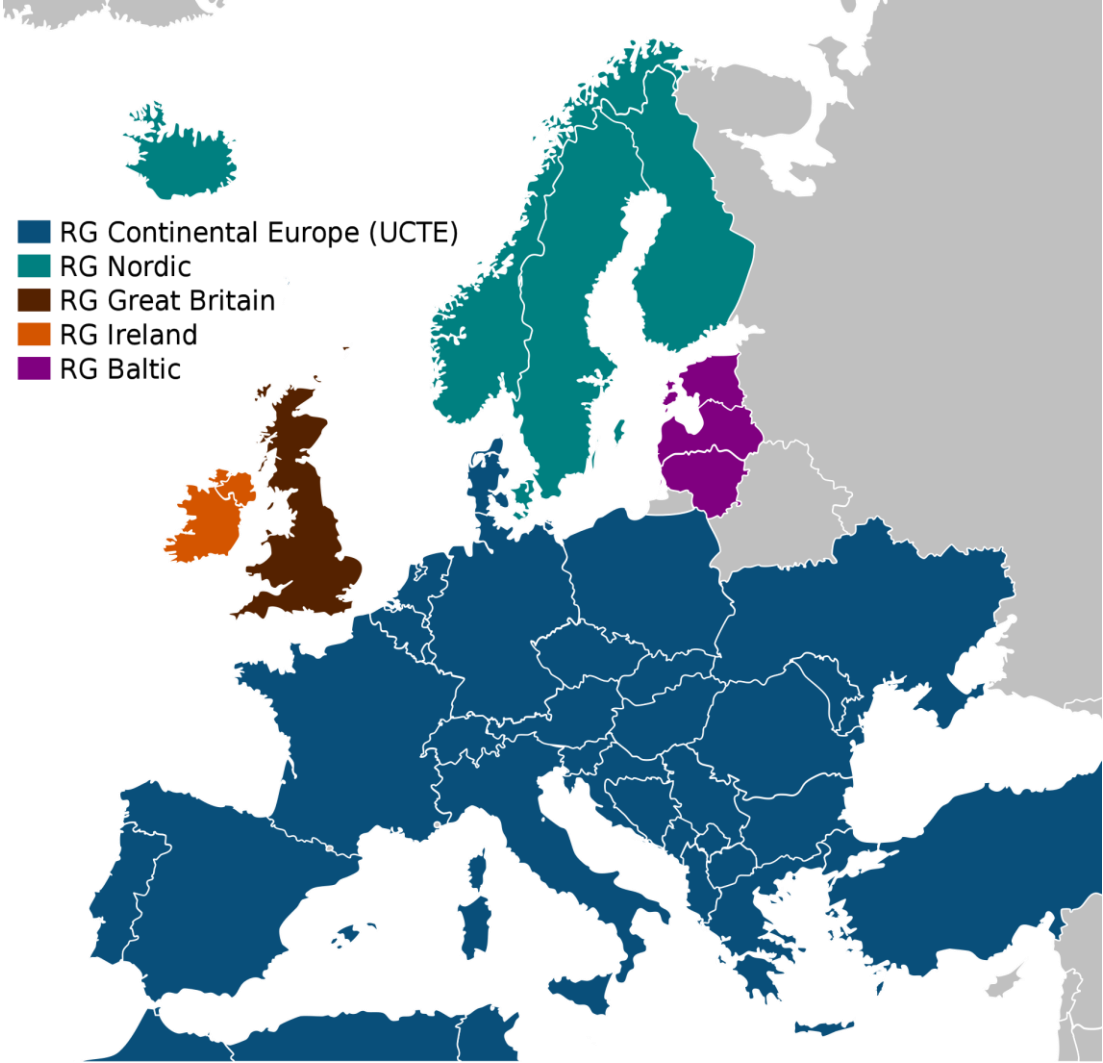
And thanks to industrial collaborators:

IBM Spirii

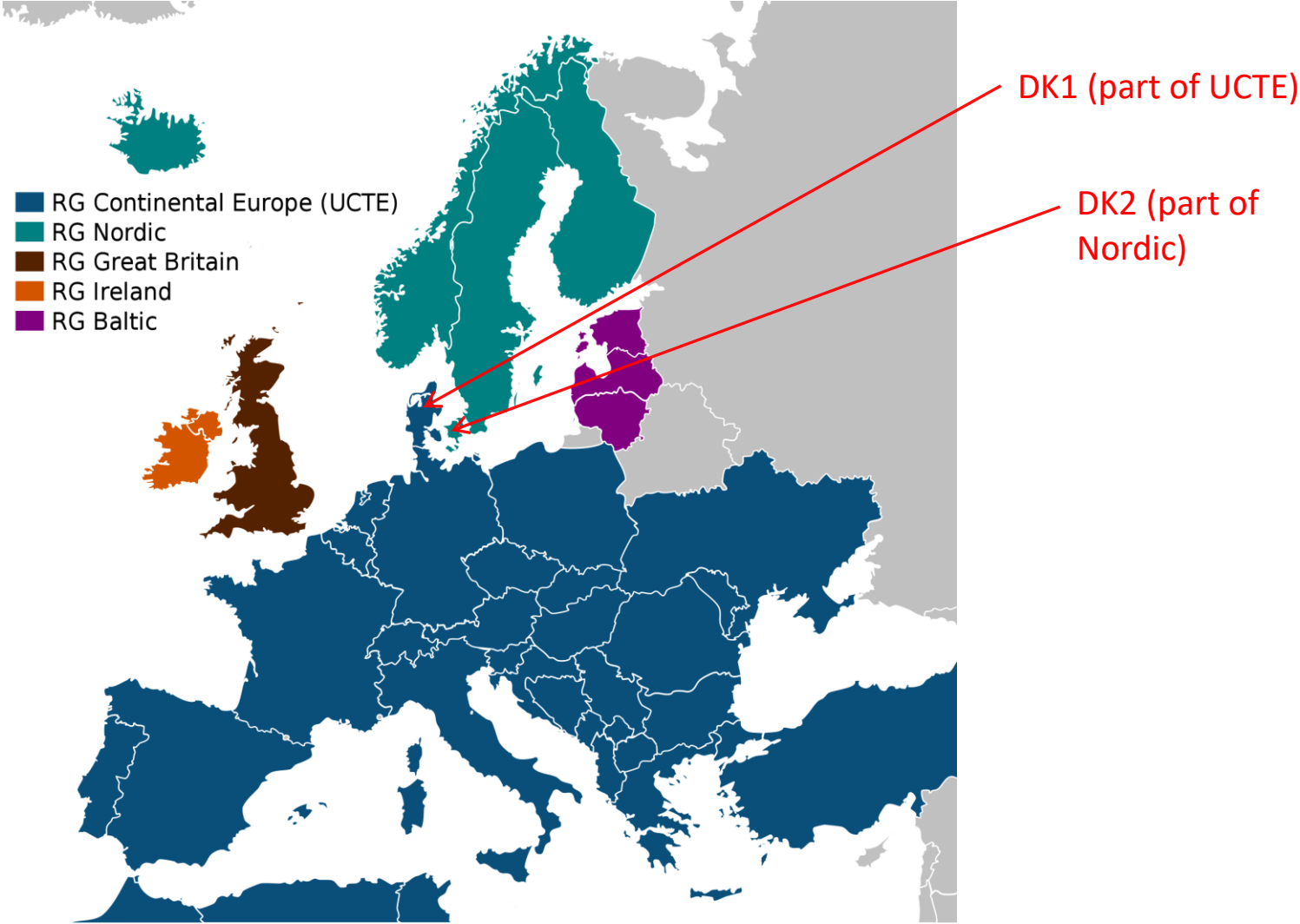
ENERGINET

Short introduction to Nordic ancillary service markets

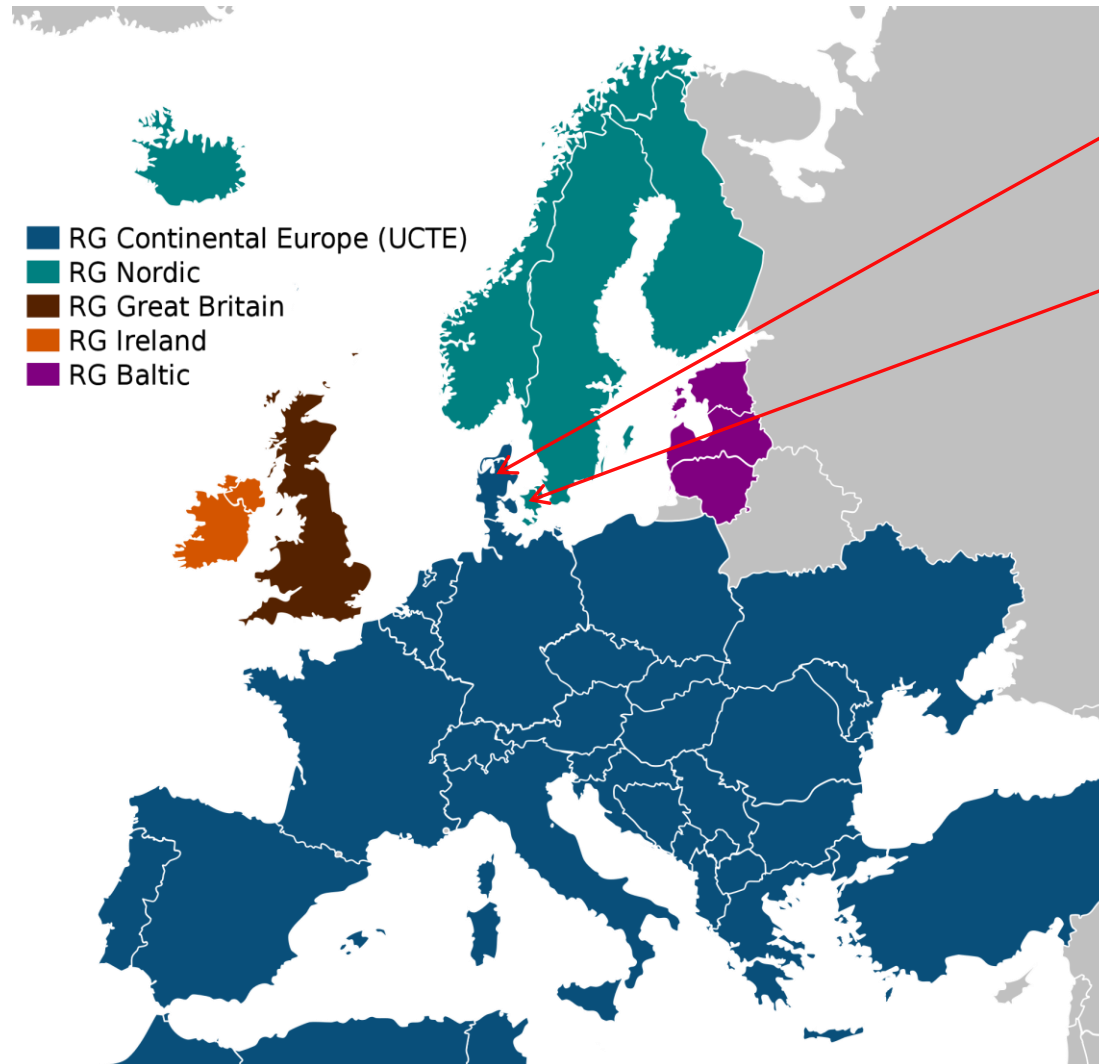
Synchronous grid areas in Europe



Synchronous grid areas in Europe



Synchronous grid areas in Europe

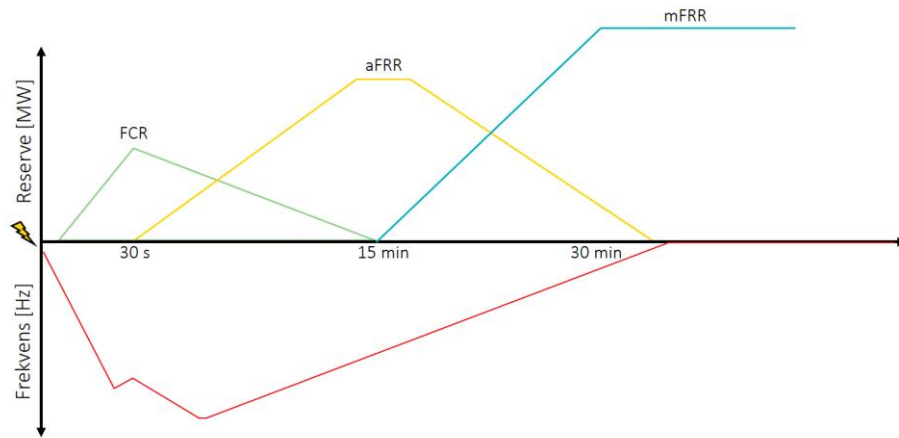


DK1 (part of UCTE)

DK2 (part of Nordic)

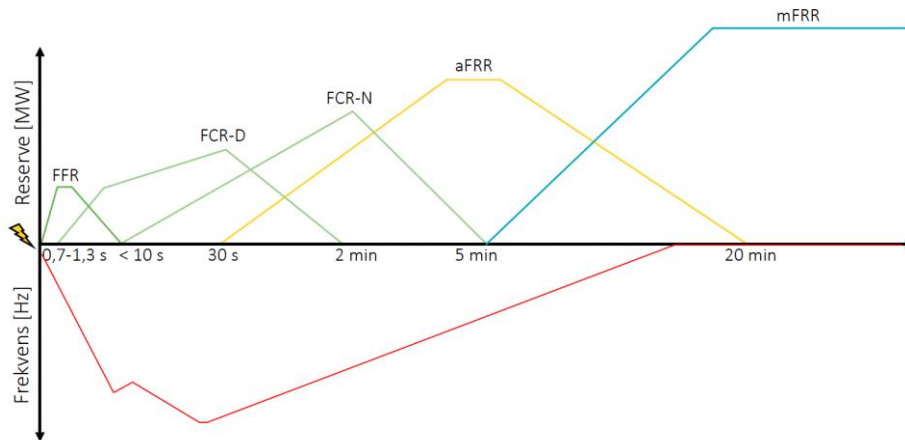
Energinet is operating the Danish power system in two areas → different ancillary services exist in DK1 and DK2

Frequency-based ancillary services in DK1 and DK2



Ancillary services in DK1 (as part of the continental area)

- FCR
- aFRR
- mFRR



Ancillary services in DK2 (as part of the Nordic area)

- FFR (fast frequency reserve)
- FCR-D (D stands for disturbance)
- FCR-N (N stands for normal)
- aFRR
- mFRR

Source: Energinet (Gennemgang af Nuværende Systemydelse Markeder)

Specifics of ancillary services in DK1 and DK2



Source: Energinet (Gennemgang af Nuværende Systemydelse Markeder)

ENERGINET

SYSTEMYDELSER: OVERSIGT 2021

DK1

DK2

- Maks tid for aktivering
- Min. leveringstid
- Min. aggregeret budstørrelse
- Gennemsnitlig historisk pris (2019-2020)

FCR

- 15-30 sek.
- 15 min.
- 1 MW
- 50-70.000 kr./MW/mdr. (rådighed)

aFFR

- 15 min. (5 min. ≈2024)
- 60 min.
- 1 MW
- 200-250.000 kr./MW/mdr. (rådighed)
- Min. spotpris + 100 kr./MWh (energi)

mFFR

- 15 min.
- 60 min. (15 min. ≈2024)
- 5 MW (1 MW ≈ 2024)
- Ca. 3-5.000 kr./MW/mdr. (rådighed)
- Op: 600 Ned: 50 kr./MWh (energi)

- 1 sek.
- 10 sek.
- 0,3 MW
- 200.000 kr./MW/mdr. (rådighed)

- 50% 5 sek. 100% 30 sek.
- 15 min.
- 0,3 MW
- 50-100.000 kr./MW/mdr. (rådighed)

- 150 sek.
- 60 min.
- 0,3 MW
- 100-150.000 kr./MW/mdr. (rådighed)

- 5 min
- 60 min
- 1 MW
- NA
- NA

- 15 min.
- 60 min. (15 min. ≈ 2024)
- 5 MW (1 MW ≈ 2024)
- 30-50.000 kr./MW/mdr. (rådighed)
- Op: 1000 Ned: 200 kr./MWh (energi)

FFR

FCR-D

FCR-N

aFFR

mFFR

Potential service providers in DK1 and DK2

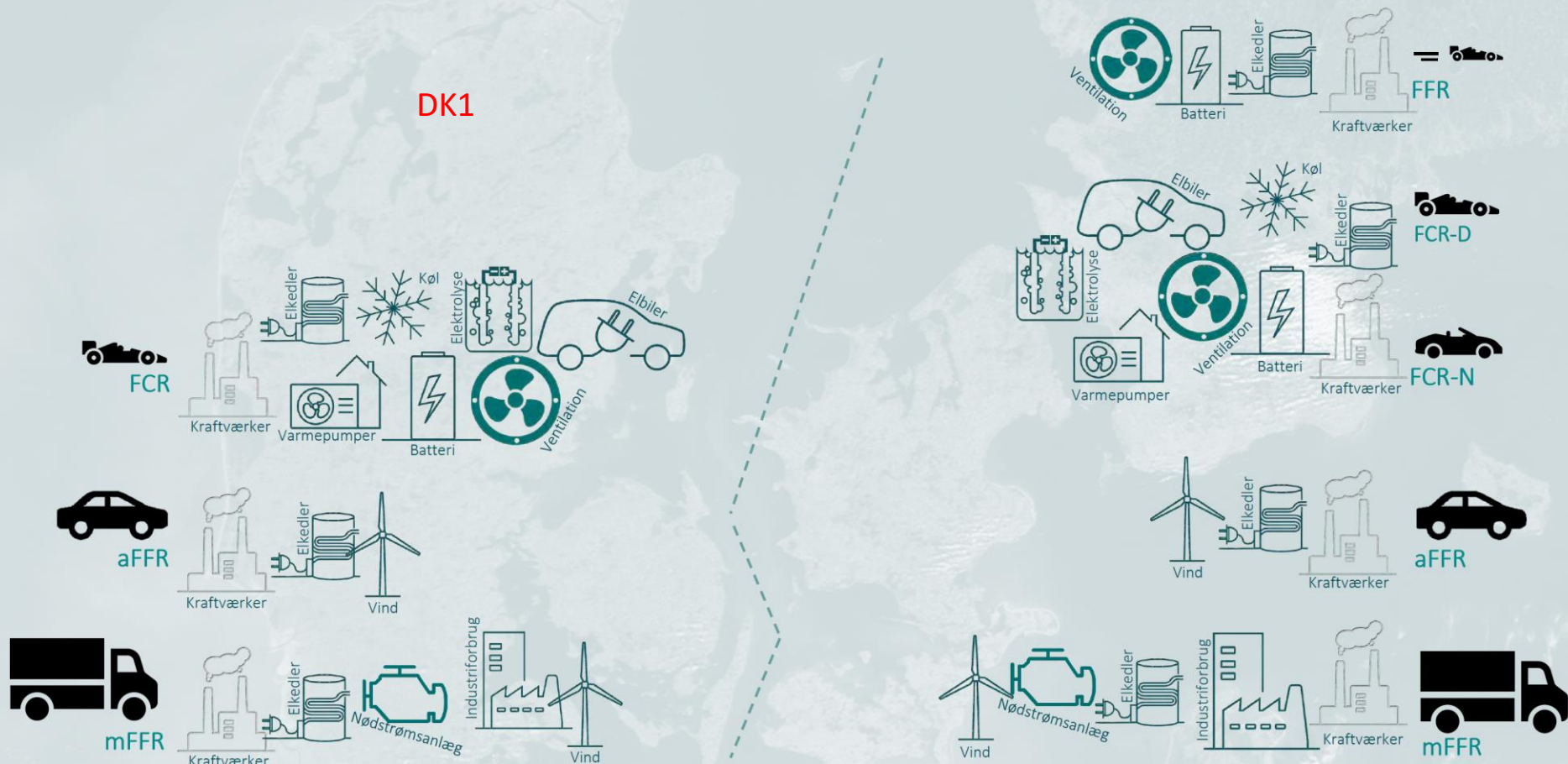


Source: Energinet (Gennemgang af Nuværende Systemydelse Markeder)

SYSTEMYDELSER: TEKNOLOGIERNE BAG

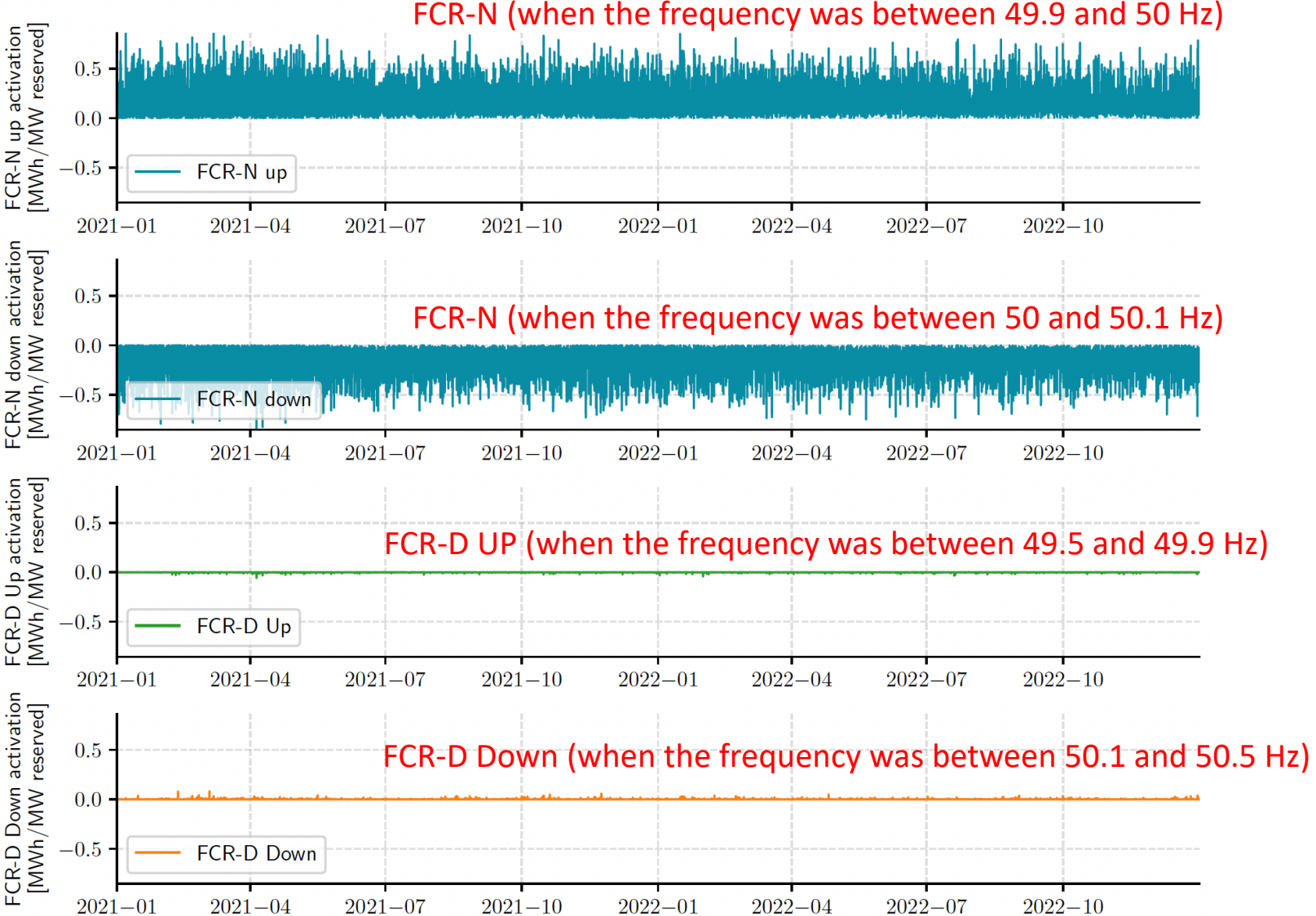
ENERGINET
DK2

DK1



Historical data: Activated FCR-D and FCR-N in DK2 (2021-2022)

Credit: Marco Saretta, DTU MSc thesis, 2023



Historical data: Activated FCR-D and FCR-N in DK2 (2021-2022)



Credit: Marco Saretta, DTU MSc thesis, 2023

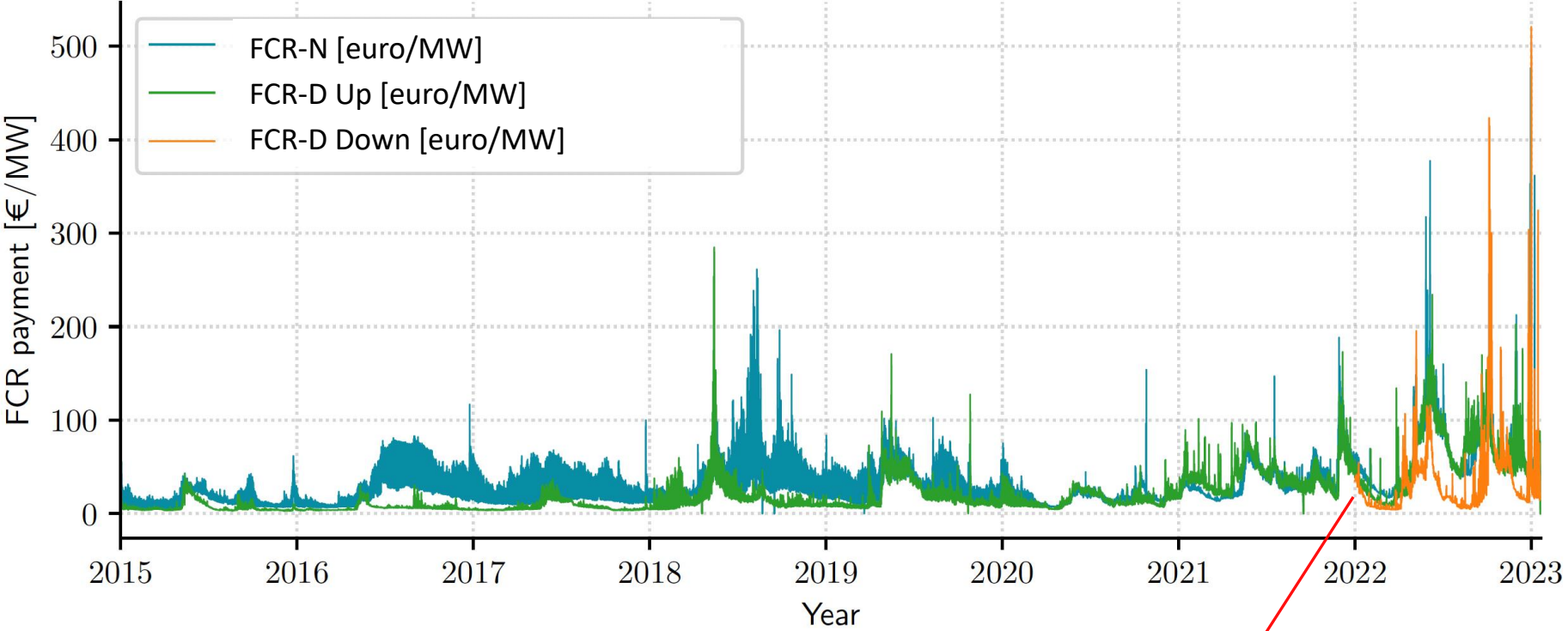


FCR-D was very rarely activated! Service providers received payments due to capacity reservation but were activated very rarely!

Historical data: FCR-D and FCR-N prices in DK2 (2015-2022)



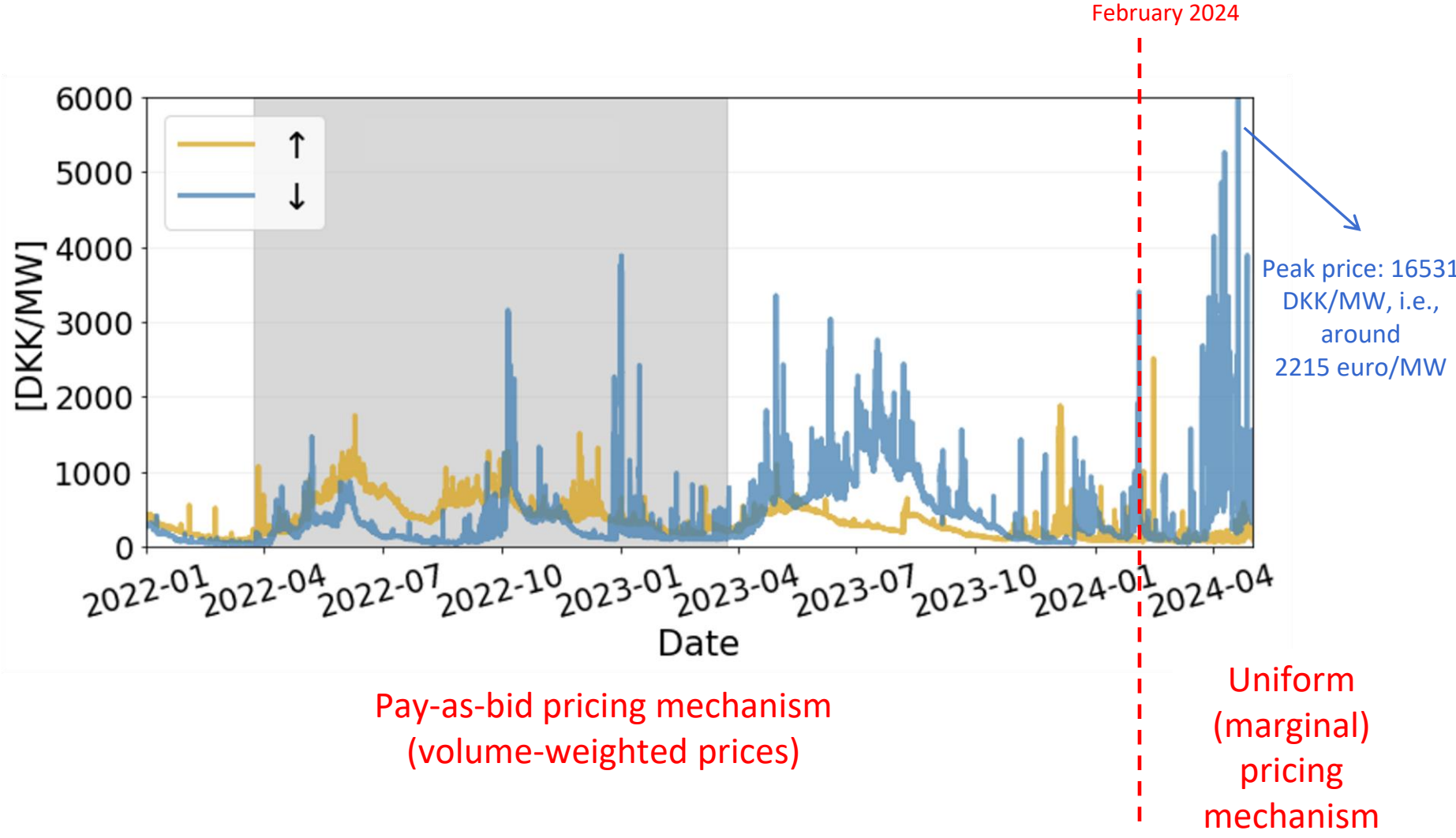
Credit: Marco Saretta, DTU MSc thesis, 2023



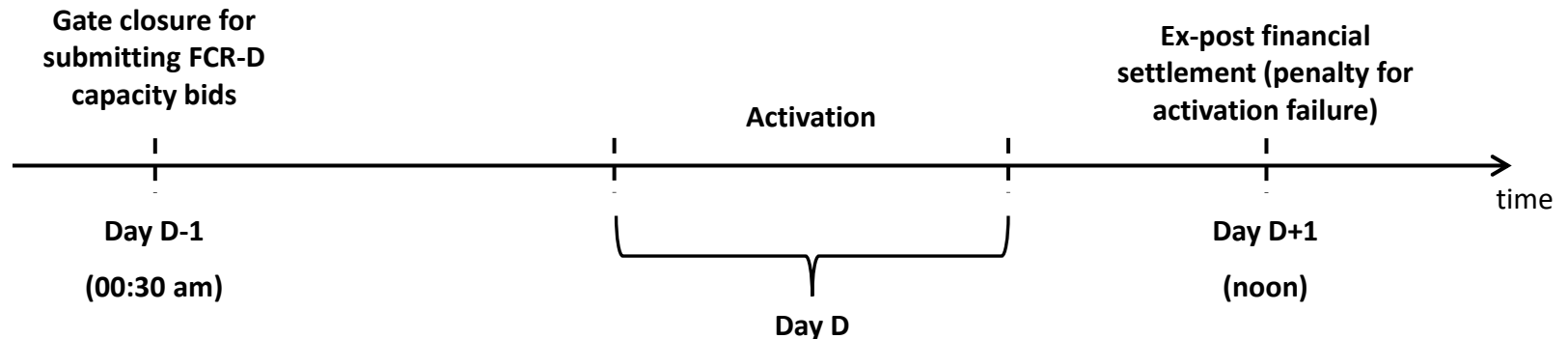
FCR-D Down started in January 2022

A closer look at historical FCR-D up/down prices in Denmark (DK2)

Credit: *Gustav Lunde and Emil Damm, DTU MSc thesis, 2024*



Current market for FCR-D Up/Down in Denmark



- The FCR-D services are used to be bought in D-2 (until very recently). Now it is in D-1.
- There is a second (optional) market for FCR-D in D-1 in case TSOs realize more FCR-D services should be bought.
- Payment for capacity only (activation is not “energy-intensive”)
- Penalty for activation failure = the cost of alternative source

Nordic TSO obligations to procure FCR services in 2023

	Share [%]	FCR-N [MW]	FCR-D Up [MW]	FCR-D Down [MW]
StatNett	39	234	564	546
FinGrid	20	120	290	280
Svenska Kraftnat	38.3	230	555	536
Energinet	2.7	17	41	38
Nordic obligations	100	600	1450	1400

Source: Energinet report [\[link\]](#)

Outlook for the need in 2030-2040: Energinet report [\[link\]](#)

Credit: Marco Saretta, DTU MSc thesis, 2023

Relevant article by Marco et al: [\[link\]](#)

Stochastic flexible assets that can bid their flexibility to ancillary service markets

- These assets could be in the demand or supply side!
- Examples of stochastic flexible assets: electric vehicles (EVs), heat pumps, supermarket freezers, wind turbines, etc.
- Future consumption/production level of these assets is stochastic → stochastic baseline!
- Without loss of generalization, from now on, we consider the FCR-D Up/Down market as our example ancillary service markets!

Data for an aggregation of EV charging boxes in Denmark

Credit: *Gustav Lunde and Emil Damm, DTU MSc thesis, 2024*



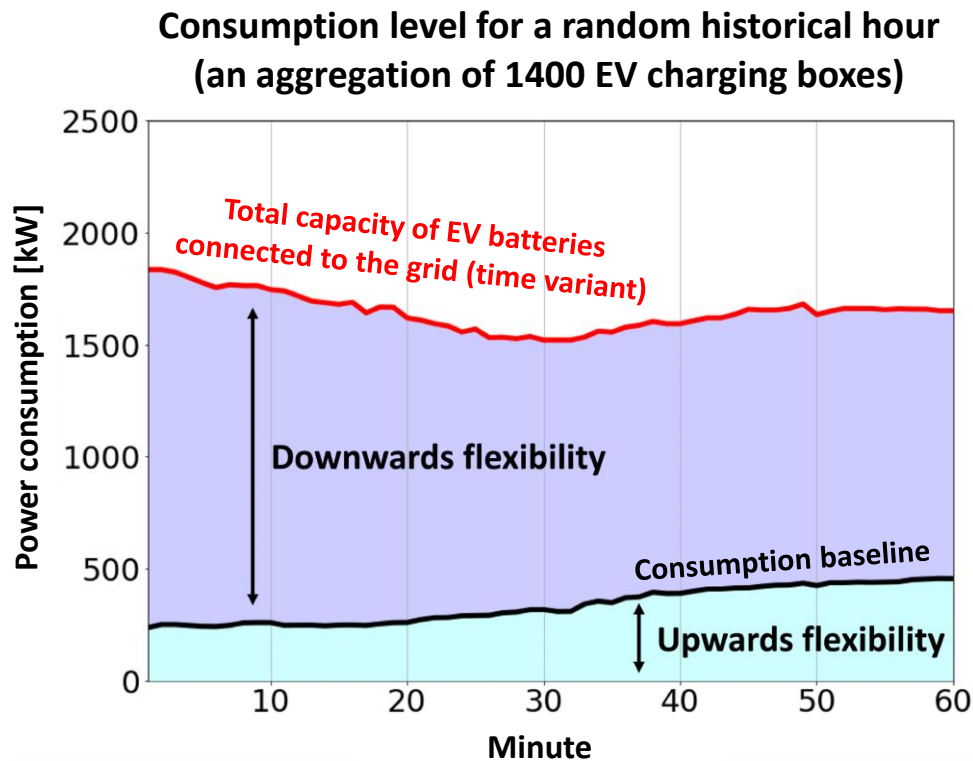
- Data for electric vehicles (EVs) provided by Spirii (<https://spirii.com/en>)
- Time period of March 24, 2022, to March 21, 2023
- Minute-level resolution (the ideal is to have a higher-resolution dataset)

Minute

Data for an aggregation of EV charging boxes in Denmark

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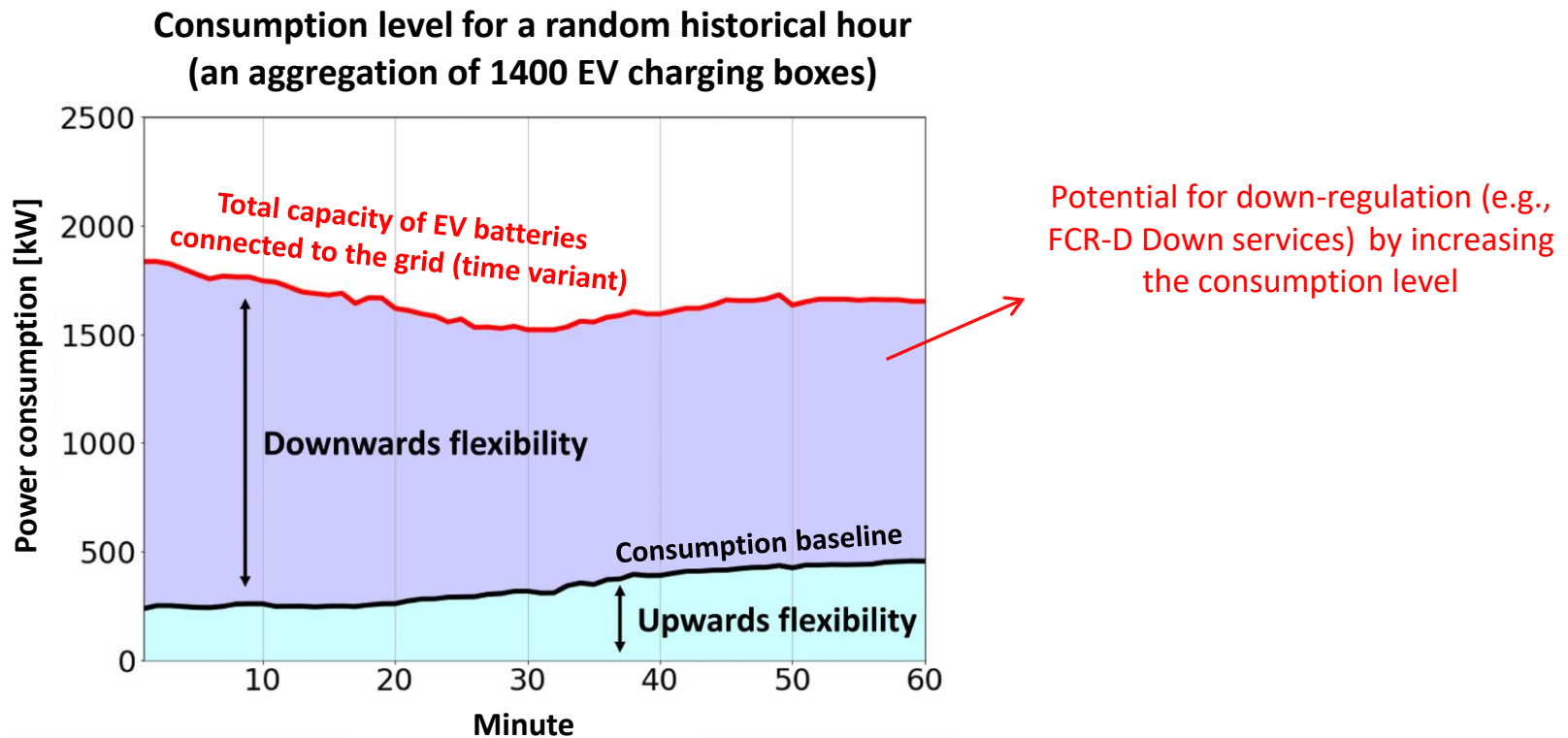
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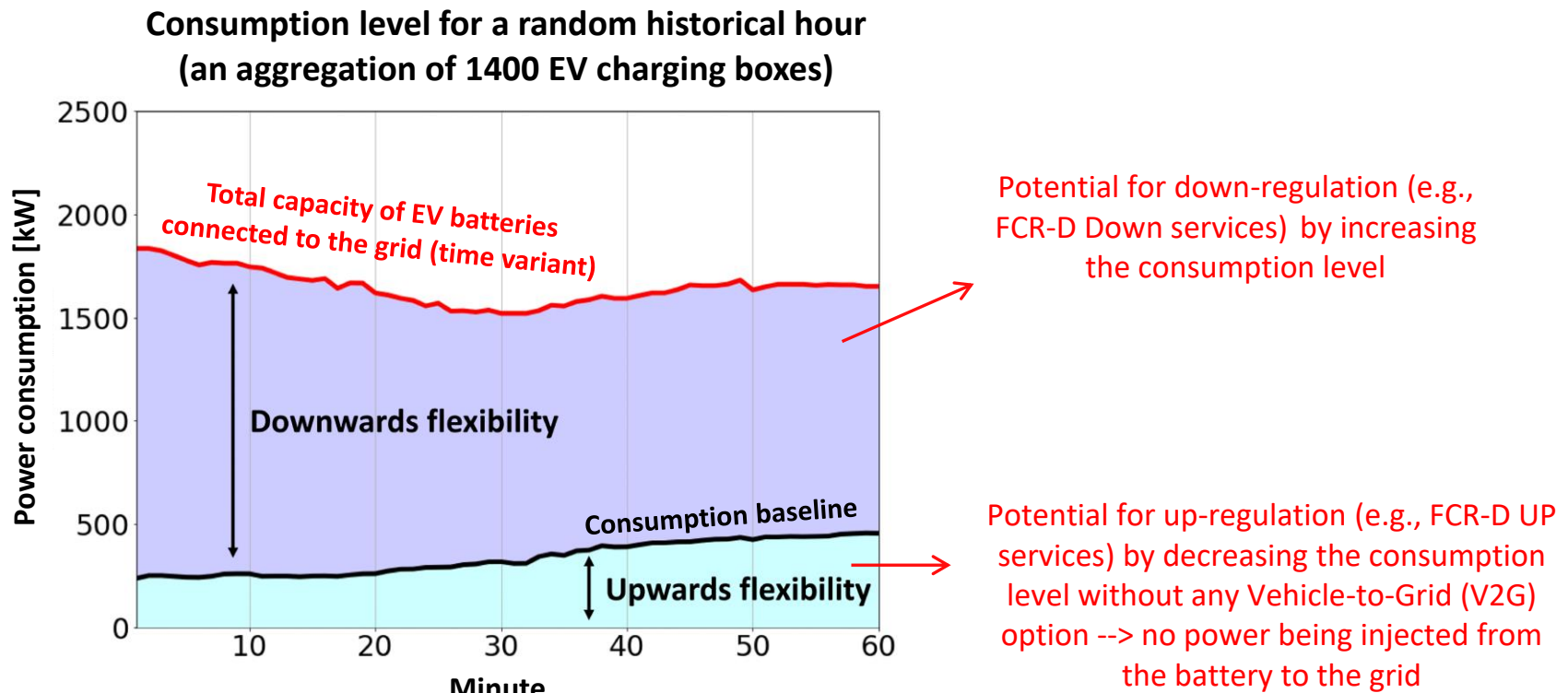
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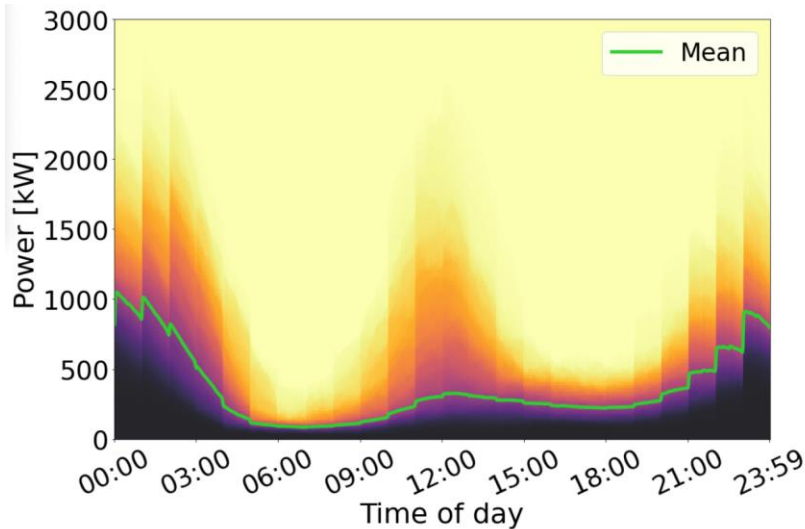
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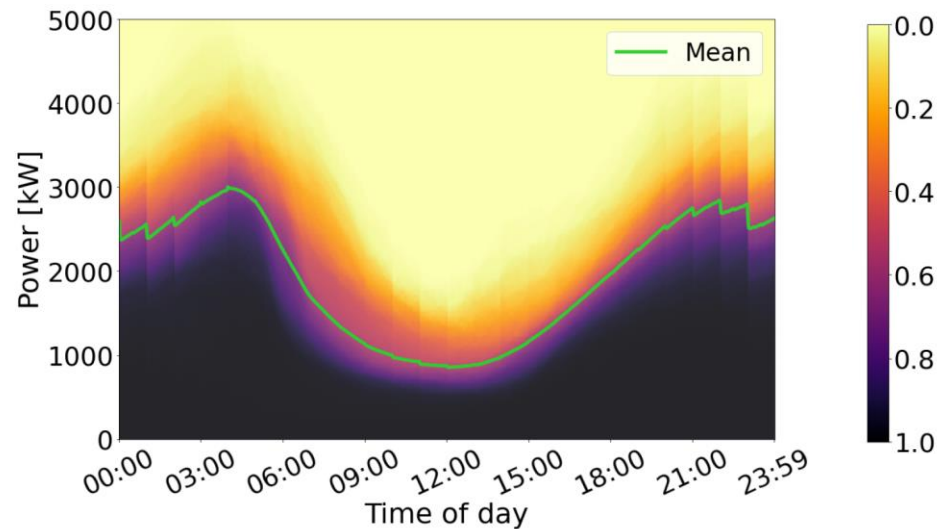
Data for an aggregation of EV charging boxes in Denmark

Credit: *Gustav Lunde and Emil Damm, DTU MSc thesis, 2024*

1-CDF of potential [kW] for **FCR-D Up (left plot)** and **FCR-D Down (right plot)** services throughout the day (based on historical data for 1400 EV charging boxes). CDF = cumulative distribution function.



(a) Upwards flexibility F^{\uparrow}

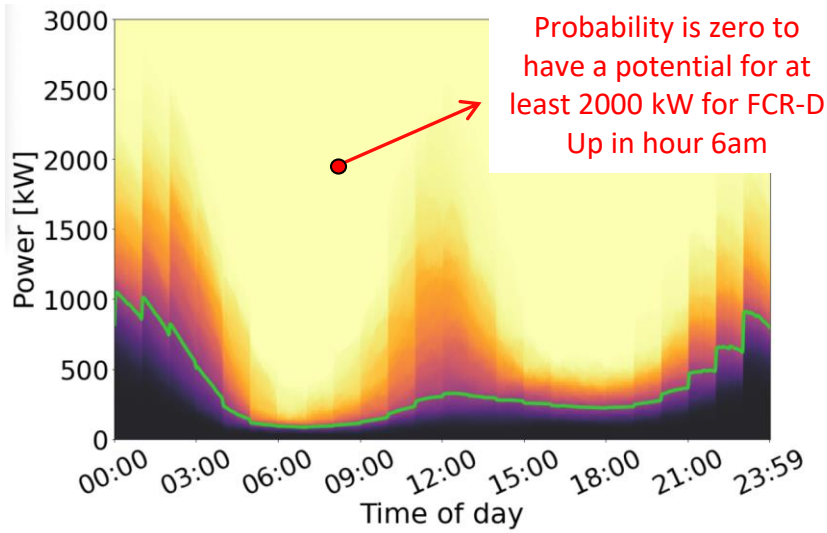


(b) Downwards flexibility F^{\downarrow}

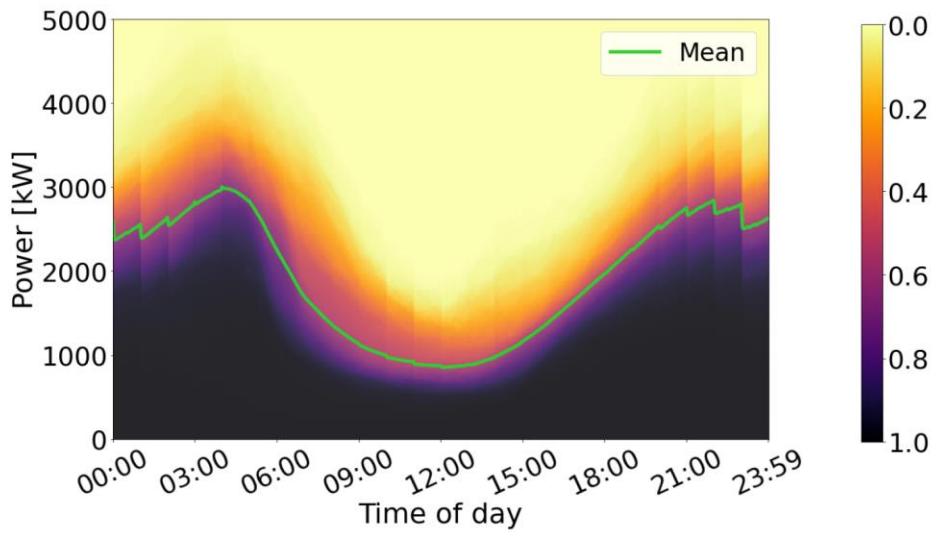
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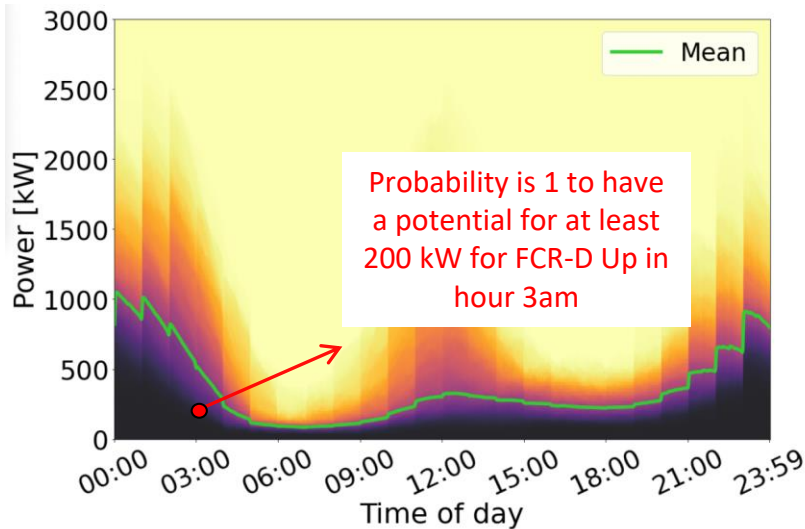


(b) Downwards flexibility F^\downarrow

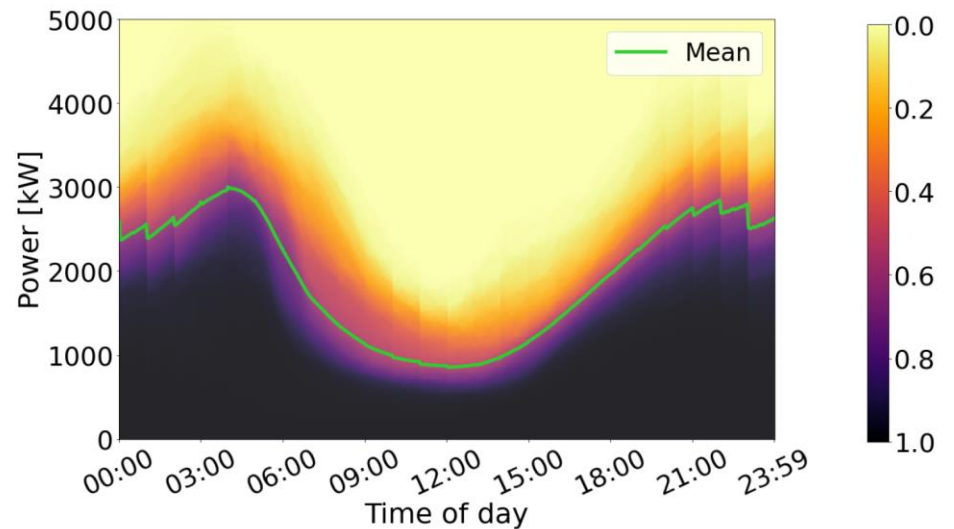
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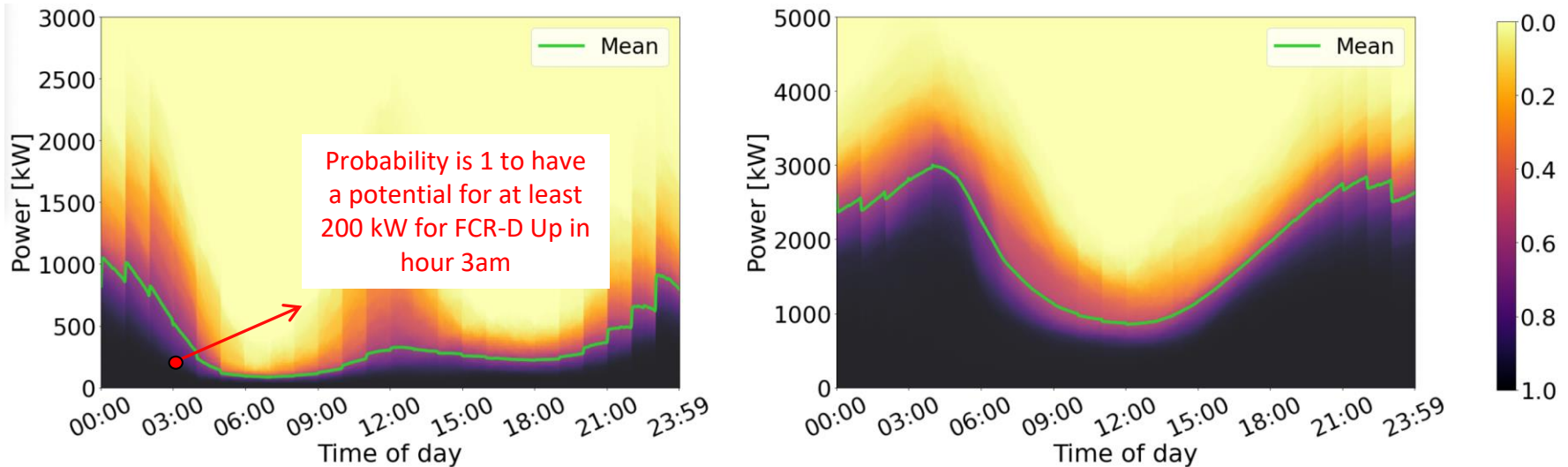


(b) Downwards flexibility F^{\downarrow}

Data for an aggregation of EV charging boxes in Denmark

Credit: *Gustav Lunde and Emil Damm, DTU MSc thesis, 2024*

1-CDF of potential [kW] for **FCR-D Up (left plot)** and **FCR-D Down (right plot)** services throughout the day (based on historical data for 1400 EV charging boxes). CDF = cumulative distribution function.



Note:

These two distribution functions are built based on available historical data during the period of March 24, 2022, to March 21, 2023. It is not necessarily the best way to utilize data. For example, one may use these data to “probabilistically forecast” the future baseline and then use it for bidding decision-making purposes. Or due to seasonality or non-stationarity reasons or alike, one may use the most recent/relevant data for the representation of baseline for the next day!

The P90 requirement of Energinet

- The name “P90” was given by us. It is not used in the Energinet report.
- “Energinet: Prequalification and test,” Energinet, 2023, accessed: 2024-05-30. [Online]. Available: <https://en.energinet.dk/electricity/ancillary-services/prequalification-and-test/>

The P90 requirement

*“Energinet requires that there must at maximum be bid in capacity corresponding to the 10% percentile with delivery of capacity reserves from fluctuating renewables and flexible consumption. This means, that the participant’s prognosis, which must be approved by Energinet, evaluates **that the probability is 10% that the sold capacity is not available. This entails that there is a 90% chance that the sold capacity or more is available.** This is when the prognosis is assumed to be correct. The probability is then also 10%, that the entire sold capacity is not available. If this were to happen, it does not entail that the sold capacity is not available at all, however just that a part of the total capacity is not available. The available part will with a high probability be close to the sold capacity.”*

Source: Energinet report [\[link\]](#)

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How do you interpret this requirement?

The P90 requirement

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Source: Energinet report [\[link\]](#)

This requirement lets a stochastic recourse bid in Nordic ancillary service markets, provided the probability of the bid to be successfully realized is at least 90% -- this means the resource will be still counted qualified for bidding in ancillary service markets if the probability of “**reserve shortfall**” (also called “**overbidding**”) is not more than 10%.

The P90 requirement

*“Energinet requires that there must at maximum be bid in capacity corresponding to the 10% percentile with delivery of capacity reserves from fluctuating renewables and flexible consumption. This means, that the participant’s prognosis, which must be approved by Energinet, evaluates **that the probability is 10% that the sold capacity is not available. This entails that there is a 90% chance that the sold capacity or more is available.** This is when the prognosis is assumed to be correct. The probability is then also 10%, that the entire sold capacity is not available. If this were to happen, it does not entail that the sold capacity is not available at all, however just that a part of the total capacity is not available. The available part will with a high probability be close to the sold capacity.”*

Source: Energinet report [\[link\]](#)

How does Energinet check this requirement for given bids?

The P90 requirement



Harry van der Weijde (He/Him) · 1st
Senior Scientist at TNO Vector | Energy | Transport | Economics | S...
1d ...

Interesting! How is the requirement enforced, since only the realisations are visible and the underlying distribution is usually not? How do bidders prove a 90% probability?

Like · 1 | Reply · 1 Reply



Thomas Dalgas Fechtenburg · 1st
Senior Manager, Ancillary Services, Energinet
1d ...

[Harry van der Weijde](#) - based on at least three months of historical performance, where the P90 proved to be available at least 15% of the time (a binary consideration). We continuously monitor the performance of both the physical delivery and forecasts as well, which allow for a "low" entry criteria.

Like · 2 | Reply



Thomas Dalgas Fechtenburg · 1st
Senior Manager, Ancillary Services, Energinet
2d ...

I'm glad you find our requirement interesting! After having it for ~3 years now, we start to see the effect of it. From our perspective it took some time to learn, but now multiple providers have developed probabilistic forecasts to meet it effectively. Looking forward to read your paper!

Like · 1 | Reply · 1 Reply



Jalal Kazempour **Author**
Head of Section, Head of Studies, Associate Professor at ...
2d ...

Thanks [Thomas](#) for the comment and all discussions we had so far. Indeed it is very innovative and interesting. I am not aware of any other TSO with a similar innovative requirement. Very nice to hear there are now some flex providers meeting

How does Energinet check it?

Mathematical representation of the P90 requirement



Let's start with the FCR-D Up market. One can formulate a similar optimization problem for the FCR-D Down market.

Bidding a reserve capacity (in kW) to the FCR-D Up market in a given hour (say hour h):

$$\text{Max}_{c^\uparrow \geq 0} c^\uparrow$$

subject to:

$$\mathbb{P} (c^\uparrow \leq F_m^\uparrow, \quad \forall m) \geq 1 - \epsilon$$

Mathematical representation of the P90 requirement

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$$\text{Max}_{c^\uparrow \geq 0} c^\uparrow$$

Reserve capacity bid [kW] in the given hour to be offered to the FCR-D Up market. This is our decision variable!

subject to:

$$\mathbb{P} (c^\uparrow \leq F_m^\uparrow, \quad \forall m) \geq 1 - \epsilon$$

Mathematical representation of the P90 requirement

Let's start with the FCR-D Up market. One can formulate a similar optimization problem for the FCR-D Down market.

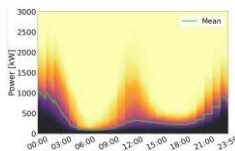
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subject to:

$$\mathbb{P}(c^\uparrow \leq F_m^\uparrow, \forall m) \geq 1 - \epsilon$$

Probability distribution of the FCR-D Up service availability per minute m of the given hour



Minutes = $\{1, 2, \dots, 60\}$ in the given hour h

Mathematical representation of the P90 requirement

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0.1 as per the P90 requirement

$\mathbb{P}(\cdot)$: Probability function.

Note that we have a “**probabilistic constraint**”!

Mathematical representation of the P90 requirement

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What does this “**probabilistic constraint**” enforce?

Mathematical representation of the P90 requirement



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What does this “**probabilistic constraint**” enforce?

It enforces the “probability” of the set of constraints inside $P(\cdot)$ to be met should be at least 90%. Given 60 minutes, it enforces our reserve capacity bid corresponding to hour h should be available at least in 54 minutes of that hour and we should not see a reserve shortfall in more than 6 minutes! It does not say about the “magnitude” of shortfall though as it is the case in the P90 requirement too.

Mathematical representation of the P90 requirement

Let's start with the FCR-D Up market. One can formulate a similar optimization problem for the FCR-D Down market.

Bidding a reserve capacity (in kW) to the FCR-D Up market in a given hour (say hour h):

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subject to:

$$\mathbb{P} \left(c^\uparrow \leq F_m^\uparrow, \quad \forall m \right) \geq 1 - \epsilon$$

What does this “**probabilistic constraint**” enforce?

- This is a “**chance-constrained program**”! It is a well-known class of optimization problems under uncertainty!
- This is specifically a “**joint**” chance-constrained program as we have more than one constraint within $\mathbb{P}(\cdot)$.

Mathematical representation of the P90 requirement

Let's start with the FCR-D Up market. One can formulate a similar optimization problem for the FCR-D Down market.

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subject to:

$$\mathbb{P} \left(c^\uparrow \leq F_m^\uparrow, \quad \forall m \right) \geq 1 - \epsilon$$

What does this “**probabilistic constraint**” enforce?

Question: Having data with a second-resolution than minutes, does it make our decision-making optimization problem more flexible (and less conservative)?

How to solve a (joint) chance-constrained program?

Two solution techniques:

1. **ALSO-X** (reference [1]-[2]. ALSO-X is the initials of co-authors in [1].)
2. **Conditional value-at-risk (CVaR) approximation**

Both techniques require **sampling** from distributions. Recall we have 60 distributions, one per minute. We draw $w = \{w_1, w_2, \dots, |w|\}$ arbitrary samples from each distribution.

[1] S. Ahmed, J. Luedtke, Y. Song, and W. Xie, “Nonanticipative duality, relaxations, and formulations for chance-constrained stochastic programs,” *Mathematical Programming*, vol. 162, no. 1, pp. 51–81, 2017.

[2] N. Jiang and W. Xie, “ALSO-X and ALSO-X+: Better convex approximations for chance constrained programs” *Operations Research*, vol. 70, no. 6, pp. 3581–3600, 2022.

How to solve a (joint) chance-constrained program?

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If the underlying probability distribution admits certain properties, we can have an “**analytical**” reformulation [3] → Satisfactory out-of-sample performance.

[3] A. Nemirovski and A. Shapiro, “Convex approximations of chance constrained programs,” *SIAM Journal on Optimization*, vol. 17, no. 4, pp. 969–996, 2007.

How to solve a (joint) chance-constrained program?

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➤ What is the minimum number of samples that we should use? See [4] for the answer!

[4] J. Luedtke and S. Ahmed, "A sample approximation approach for optimization with probabilistic constraints," *SIAM Journal of Optimization*, vol. 19, no. 2, pp. 674-699, 2008.

ALSO-X

Chance constraint → sample-based MILP reformulation → LP relaxation → iterative algorithm

ALSO-X solution technique

$$\begin{aligned} & \text{Max} \quad c^\uparrow \\ & c^\uparrow \geq 0 \\ & \text{subject to:} \\ & \mathbb{P} \left(c^\uparrow \leq F_m^\uparrow, \quad \forall m \right) \geq 1 - \epsilon \end{aligned}$$

Joint chance-constrained program

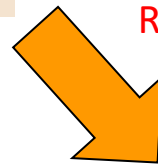
ALSO-X solution technique

$$\text{Max}_{c^\uparrow \geq 0} c^\uparrow$$

subject to:

$$\mathbb{P} (c^\uparrow \leq F_m^\uparrow, \quad \forall m) \geq 1 - \epsilon$$

Joint chance-constrained program

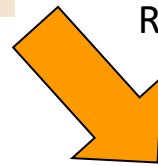


Reformulation based on samples

ALSO-X solution technique

$$\begin{aligned} & \text{Max} \quad c^\uparrow \\ & c^\uparrow \geq 0 \\ & \text{subject to:} \\ & \mathbb{P} \left(c^\uparrow \leq F_m^\uparrow, \quad \forall m \right) \geq 1 - \epsilon \end{aligned}$$

Joint chance-constrained program



Reformulation based on samples

$$\begin{aligned} & \text{Max} \quad c^\uparrow \\ & c^\uparrow \geq 0, y_{m,\omega} \in \{0,1\} \\ & \text{subject to:} \\ & c^\uparrow - F_{m,\omega}^\uparrow \leq y_{m,\omega} M \quad \forall m, \omega \\ & \sum_m \sum_\omega y_{m,\omega} \leq q \end{aligned}$$

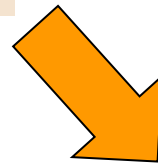
Sample-based mixed-integer linear program (MILP)

ALSO-X solution technique

$$\text{Max}_{c^\uparrow \geq 0} c^\uparrow$$

subject to:

$$\mathbb{P}(c^\uparrow \leq F_m^\uparrow, \forall m) \geq 1 - \epsilon$$



Unchanged.
Not indexed by sample w .

$$\text{Max}_{c^\uparrow \geq 0, y_{m,\omega} \in \{0,1\}} \boxed{c^\uparrow}$$

subject to:

$$c^\uparrow - F_{m,\omega}^\uparrow \leq y_{m,\omega} M \quad \forall m, \omega$$

$$\sum_m \sum_\omega y_{m,\omega} \leq q$$

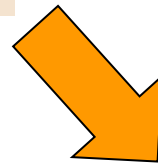
ALSO-X solution technique

$$\text{Max } c^\uparrow$$

$$c^\uparrow \geq 0$$

subject to:

$$\mathbb{P}(c^\uparrow \leq F_m^\uparrow, \quad \forall m) \geq 1 - \epsilon$$



$$\text{Max } c^\uparrow$$

$$c^\uparrow \geq 0, y_{m,\omega} \in \{0,1\}$$

Binary variables, one per minute, per sample!

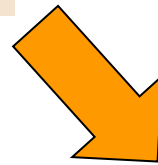
subject to:

$$c^\uparrow - F_{m,\omega}^\uparrow \leq y_{m,\omega} M \quad \forall m, \omega$$

$$\sum_m \sum_\omega y_{m,\omega} \leq q$$

ALSO-X solution technique

$$\begin{aligned} & \text{Max } c^\uparrow \\ & c^\uparrow \geq 0 \\ & \text{subject to:} \\ & \mathbb{P} (c^\uparrow \leq F_m^\uparrow, \quad \forall m) \geq 1 - \epsilon \end{aligned}$$



$$\begin{aligned} & \text{Max } c^\uparrow \\ & c^\uparrow \geq 0, y_{m,\omega} \in \{0,1\} \\ & \text{subject to: } \text{Sample } w \text{ from the} \\ & \quad \text{distribution for minute } m \\ & c^\uparrow - F_{m,\omega}^\uparrow \leq y_{m,\omega} M \quad \forall m, \omega \\ & \sum_m \sum_\omega y_{m,\omega} \leq q \end{aligned}$$

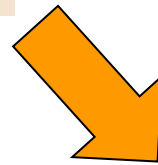
ALSO-X solution technique

$$\text{Max } c^\uparrow$$

$$c^\uparrow \geq 0$$

subject to:

$$\mathbb{P} (c^\uparrow \leq F_m^\uparrow, \quad \forall m) \geq 1 - \epsilon$$



$$\text{Max } c^\uparrow$$

$$c^\uparrow \geq 0, y_{m,\omega} \in \{0,1\}$$

subject to:

$$c^\uparrow - F_{m,\omega}^\uparrow \leq y_{m,\omega} \boxed{M} \quad \forall m, \omega$$

$$\sum_m \sum_\omega y_{m,\omega} \leq q$$

A large enough positive constant, e.g., 10000.

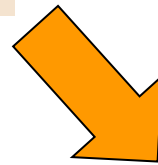
ALSO-X solution technique

$$\text{Max } c^\uparrow$$

$$c^\uparrow \geq 0$$

subject to:

$$\mathbb{P}(c^\uparrow \leq F_m^\uparrow, \forall m) \geq 1 - \epsilon$$



Indicating whether the probabilistic constraint in the original problem has been “**violated**” in minute m under sample w :

$y=0 \rightarrow$ no
 $y=1 \rightarrow$ yes

$$\text{Max } c^\uparrow$$

$$c^\uparrow \geq 0, y_{m,\omega} \in \{0,1\}$$

subject to:

$$c^\uparrow - F_{m,\omega}^\uparrow \leq y_{m,\omega} M \quad \forall m, \omega$$

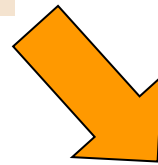
$$\sum_m \sum_\omega y_{m,\omega} \leq q$$

ALSO-X solution technique

$$\text{Max}_{c^\uparrow \geq 0} c^\uparrow$$

subject to:

$$\mathbb{P}(c^\uparrow \leq F_m^\uparrow, \forall m) \geq 1 - \epsilon$$



$$\text{Max}_{c^\uparrow \geq 0, y_{m,\omega} \in \{0,1\}} c^\uparrow$$

subject to:

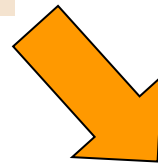
$$c^\uparrow - F_{m,\omega}^\uparrow \leq y_{m,\omega} M \quad \forall m, \omega$$

$$\sum_m \sum_\omega y_{m,\omega} \leq q$$

Total cases (for all minutes and samples) the original probabilistic constraint has been **“violated”!**

ALSO-X solution technique

$$\begin{aligned} & \text{Max } c^\uparrow \\ & c^\uparrow \geq 0 \\ & \text{subject to:} \\ & \mathbb{P} (c^\uparrow \leq F_m^\uparrow, \quad \forall m) \geq 1 - \epsilon \end{aligned}$$



$$\begin{aligned} & \text{Max } c^\uparrow \\ & c^\uparrow \geq 0, y_{m,\omega} \in \{0,1\} \\ & \text{subject to:} \\ & c^\uparrow - F_{m,\omega}^\uparrow \leq y_{m,\omega} M \quad \forall m, \omega \\ & \sum_m \sum_\omega y_{m,\omega} \leq q \end{aligned}$$

Our budget for violation (given parameter)
= 10% * number of samples * number of minutes

ALSO-X solution technique

The challenge of this problem is the number of binary variables. With 1000 samples, we will have 60,000 binary variables, which may make the problem **computationally expensive or even intractable!**

$$\begin{aligned}
 & \text{Max} && c^\uparrow \\
 & c^\uparrow \geq 0, y_{m,\omega} \in \{0,1\} \\
 & \text{subject to:} \\
 & c^\uparrow - F_{m,\omega}^\uparrow \leq y_{m,\omega} M \quad \forall m, \omega \\
 & \sum_m \sum_\omega y_{m,\omega} \leq q
 \end{aligned}$$

MILP

ALSO-X solution technique

Algorithm 1 ALSO-X

Input: Stopping tolerance parameter δ , e.g., $\delta = 10^{-5}$

Require: Relax the integrality of y

- 1: $\underline{q} \leftarrow 0$
- $\bar{q} \leftarrow \epsilon \times \text{number of samples} \times \text{number of samples}$
- 2: **while** $\bar{q} - \underline{q} \geq \delta$ **do**
- 3: Set $q = \frac{(\underline{q} + \bar{q})}{2}$
- 4: Retrieve Θ^* as an optimal solution to the relaxed problem, i.e., the LP.
- 5: Set $\underline{q} = q$ if $\mathbb{P}(y_{m,\omega}^* = 0) \geq 1 - \epsilon$; otherwise, $\bar{q} = q$
- 6: **end while**

Output: A feasible solution to the non-relaxed problem, i.e., the MILP.

Let's **relax** every binary variable between zero and one (so, MILP \rightarrow LP) and solve an **iterative** algorithm the so-called ALSO-X algorithm!



$$\begin{aligned}
 & \text{Max} && c^\uparrow \\
 & c^\uparrow \geq 0, y_{m,\omega} \in \{0,1\} \\
 & \text{subject to:} \\
 & c^\uparrow - F_{m,\omega}^\uparrow \leq y_{m,\omega} M \quad \forall m,\omega \\
 & \sum_m \sum_\omega y_{m,\omega} \leq q
 \end{aligned}$$

MILP

CVaR

Chance constraint \rightarrow CVaR constraint (conservative approximation of chance constraint) \rightarrow sample-based convex reformulation

CVaR reformulation

- The CVaR method [4] **approximates** the joint chance constraint by controlling **magnitude** of reserve shortfall using a reformulated LP. This is why the CVaR reformulation is more conservative than the original chance-constrained problem.

[4] R. T. Rockafellar and S. Uryasev, "Optimization of conditional value-at-risk," *Journal of Risk*, vol. 2, pp. 21–42, 2000

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- Thus, the CVaR minimizes the expected reserve shortfall for the worst $(1-\epsilon)$ samples which is the value-at-risk (VaR). Recall $\epsilon = 0.1$ as per the P90 requirement.

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- Thus, the CVaR minimizes the expected reserve shortfall for the worst $(1-\epsilon)$ samples which is the value-at-risk (VaR). Recall $\epsilon = 0.1$ as per the P90 requirement.
- The CVaR approximation problem reads as

$$\text{Max}_{c^\uparrow \geq 0, \beta \leq 0, \zeta_{m,\omega}} c^\uparrow$$

subject to:

$$c^\uparrow - F_{m,\omega}^\uparrow \leq \zeta_{m,\omega} \quad \forall m, \omega$$

$$\frac{1}{|m||\omega|} \sum_m \sum_\omega \zeta_{m,\omega} \leq (1 - \epsilon)\beta$$

$$\beta \leq \zeta_{m,\omega} \quad \forall m, \omega$$

[4] R. T. Rockafellar and S. Uryasev, "Optimization of conditional value-at-risk," *Journal of Risk*, vol. 2, pp. 21–42, 2000

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$$\text{Max}_{c^\uparrow \geq 0, \beta \leq 0, \zeta_{m,\omega}} c^\uparrow$$

subject to:

$$c^\uparrow - F_{m,\omega}^\uparrow \leq \zeta_{m,\omega} \quad \forall m, \omega$$

number of samples *
number of minutes

$$\frac{1}{|m||\omega|} \sum_m \sum_\omega \zeta_{m,\omega} \leq (1 - \epsilon)\beta$$

$$\beta \leq \zeta_{m,\omega} \quad \forall m, \omega$$

[4] R. T. Rockafellar and S. Uryasev, "Optimization of conditional value-at-risk," *Journal of Risk*, vol. 2, pp. 21–42, 2000

Further requirements of Energinet

LER requirement of Energinet

Source: Energinet report [[link](#)]

*“There are additional requirements for units and portfolios with **limited energy reservoir (LER)** units, such as **batteries**.”*

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*“There are additional requirements for units and portfolios with **limited energy reservoir (LER)** units, such as **batteries**.”*

Example: LER requirement for FCR-D Up in DK2:

“If you wish to prequalify 1 MW for FCR-D upwards, you must reserve 0.2 MW in the downwards direction for Normal State Energy Management (NEM) as well as 20 minutes of full FCR-D upwards delivery, or 0.33 MWh of energy.”

LER requirement of Energinet

Source: Energinet report [[link](#)]

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How do you interpret this requirement?

LER requirement of Energinet

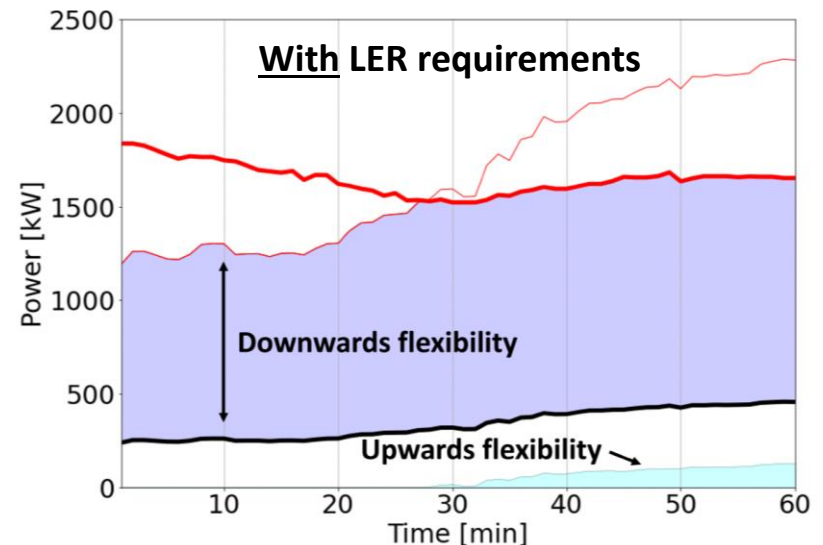
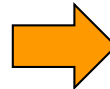
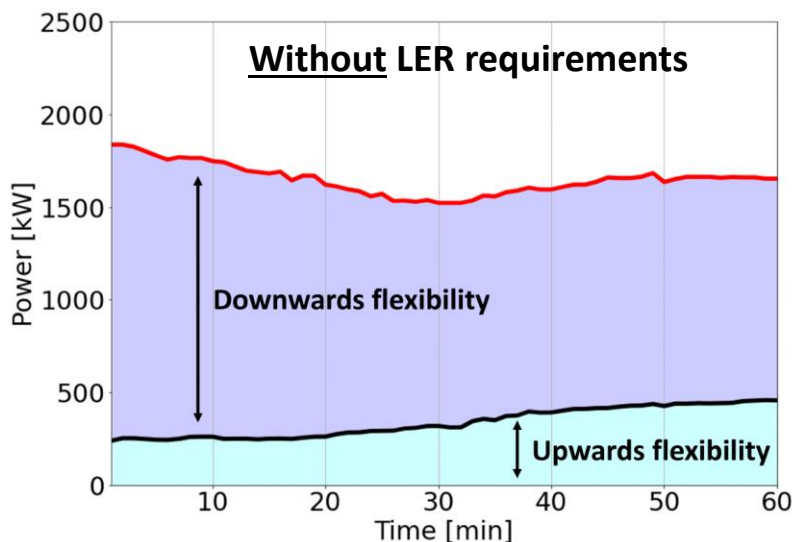
Source: Energinet report [\[link\]](#)

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Consumption level for a random historical hour (an aggregation of 1400 EV charging boxes).



Revisited chance-constrained program with the LER requirement

For each hour h :

$$\begin{aligned} & \text{Maximize} && c_h^\downarrow + c_h^\uparrow \\ & c_h^\downarrow \geq 0, c_h^\uparrow \geq 0 \end{aligned}$$

s.t.

$$\mathbb{P} \left(\begin{array}{l} \frac{1}{5} c_h^\downarrow + c_h^\uparrow \leq F_{m,h}^\uparrow \\ c_h^\downarrow \leq F_{m,h}^\downarrow \\ c_h^\downarrow \leq F_{m,h}^E \end{array} \quad \begin{array}{l} \forall m \\ \forall m \\ \forall m \end{array} \right) \geq 1 - \epsilon \quad \forall h$$

Revisited chance-constrained program with the LER requirement

For each hour h :

$$\text{Maximize } c_h^\downarrow + c_h^\uparrow$$

FCR-D Down bid [kW]
FCR-D Up bid [kW]

$c_h^\downarrow \geq 0, c_h^\uparrow \geq 0$

s.t.

$$\mathbb{P} \left(\begin{array}{l} \frac{1}{5} c_h^\downarrow + c_h^\uparrow \leq F_{m,h}^\uparrow \\ c_h^\downarrow \leq F_{m,h}^\downarrow \\ c_h^\downarrow \leq F_{m,h}^E \end{array} \quad \begin{array}{l} \forall m \\ \forall m \\ \forall m \end{array} \right) \geq 1 - \epsilon \quad \forall h$$

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For each hour h :

$$\text{Maximize } c_h^\downarrow + c_h^\uparrow$$

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s.t.

$$\mathbb{P} \left(\begin{array}{l} \frac{1}{5} c_h^\downarrow + c_h^\uparrow \leq F_{m,h}^\uparrow \\ c_h^\downarrow \leq F_{m,h}^\downarrow \\ c_h^\downarrow \leq F_{m,h}^E \end{array} \quad \begin{array}{l} \forall m \\ \forall m \\ \forall m \end{array} \right) \geq 1 - \epsilon \quad \forall h$$

Probability distribution of upward/downward flexibility availability in minute m

Revisited chance-constrained program with the LER requirement

For each hour h :

$$\text{Maximize } c_h^\downarrow + c_h^\uparrow$$

$$c_h^\downarrow \geq 0, c_h^\uparrow \geq 0$$

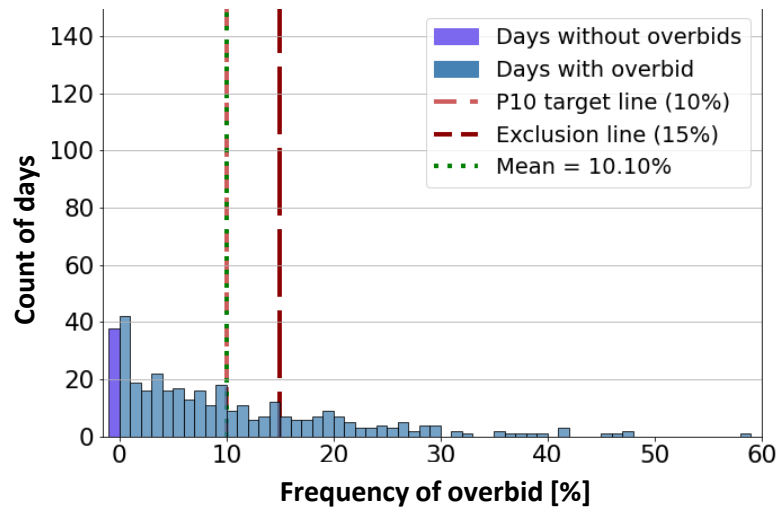
s.t.

$$\mathbb{P} \left(\begin{array}{l} \frac{1}{5} c_h^\downarrow + c_h^\uparrow \leq F_{m,h}^\uparrow \\ c_h^\downarrow \leq F_{m,h}^\downarrow \\ c_h^\downarrow \leq F_{m,h}^E \end{array} \quad \begin{array}{l} \forall m \\ \forall m \\ \forall m \end{array} \right) \geq 1 - \epsilon \quad \forall h$$

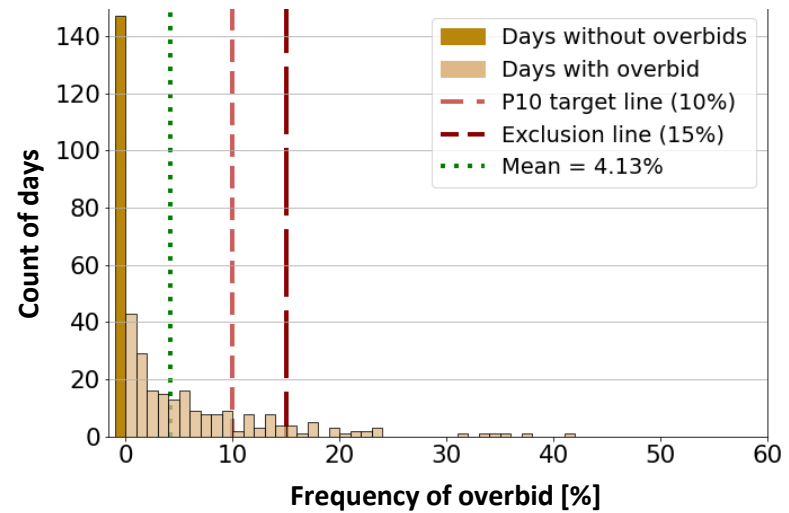
Probability distribution of downward flexibility availability in minute m such that aggregated battery can be charged, if the service fully activated, for the next 20 minutes

Out-of-sample results over a year

ALSO-X

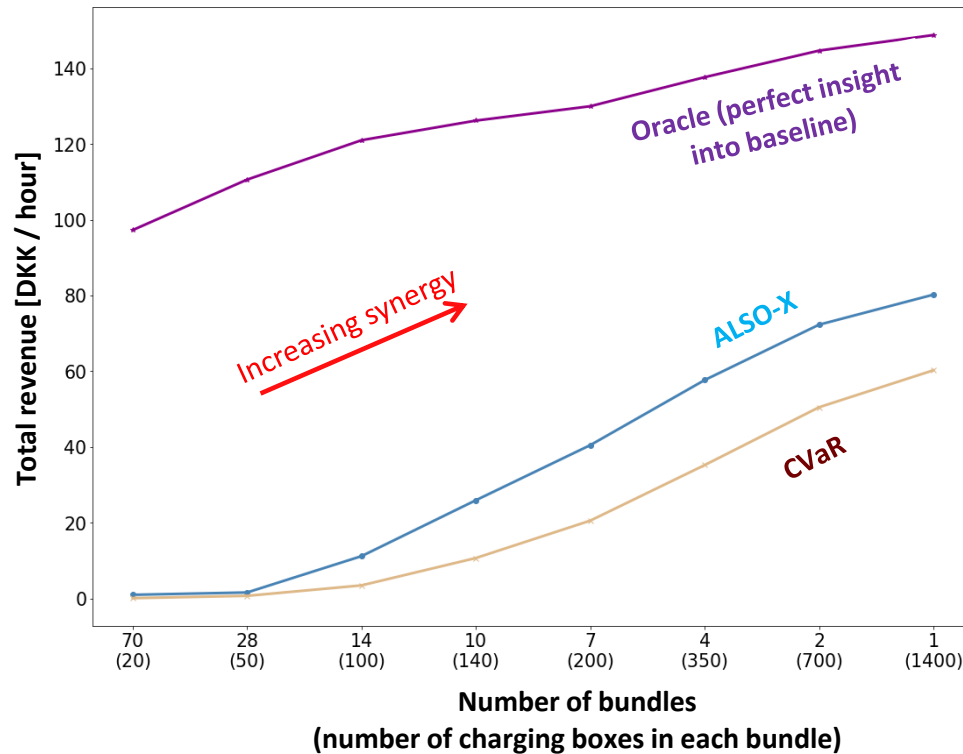


CVaR



Out-of-sample results over a year

Total profit (median) of 1400 charging boxes per hour:



Towards distributional robustness

Wasserstein distributionally robust joint chance-constrained optimization
(uncertainty in the right-hand side):

$$\text{Maximize } c_h^\downarrow + c_h^\uparrow$$

$$c_h^\downarrow \geq 0, c_h^\uparrow \geq 0$$

s.t.

$$\min_{\mathbb{P} \in \mathcal{P}} \mathbb{P} \left(\begin{array}{l} \frac{1}{5} c_h^\downarrow + c_h^\uparrow \leq F_{m,h}^\uparrow \\ c_h^\downarrow \leq F_{m,h}^\downarrow \\ c_h^\downarrow \leq F_{m,h}^E \end{array} \quad \begin{array}{l} \forall m \\ \forall m \\ \forall m \end{array} \right) \geq 1 - \epsilon \quad \forall h$$

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(uncertainty in the right-hand side):

$$\text{Maximize } c_h^\downarrow + c_h^\uparrow$$

$$c_h^\downarrow \geq 0, c_h^\uparrow \geq 0$$

s.t.

$$\min_{\mathbb{P} \in \mathcal{P}} \mathbb{P} \left(\begin{array}{l} \frac{1}{5} c_h^\downarrow + c_h^\uparrow \leq F_{m,h}^\uparrow \\ c_h^\downarrow \leq F_{m,h}^\downarrow \\ c_h^\downarrow \leq F_{m,h}^E \end{array} \quad \begin{array}{l} \forall m \\ \forall m \\ \forall m \end{array} \right) \geq 1 - \epsilon \quad \forall h$$

where

$$\mathcal{P} = \left\{ \mathbb{P} : d_W(\mathbb{P}, \hat{\mathbb{P}}_N) \leq \theta \right\}.$$

Wasserstein distance Empirical distribution

Radius (given)

Towards distributional robustness

We adopt Proposition 2 of [5] for an exact reformulation of the joint chance constraint:

PROPOSITION 2. For the safety set $\mathcal{S}(\mathbf{x}) = \{\boldsymbol{\xi} \in \mathbb{R}^K \mid \mathbf{a}_m^\top \mathbf{x} < \mathbf{b}_m^\top \boldsymbol{\xi} + b_m \ \forall m \in [M]\}$, where $\mathbf{b}_m \neq \mathbf{0}$ for all $m \in [M]$, the chance constrained program (2) is equivalent to the mixed integer conic program

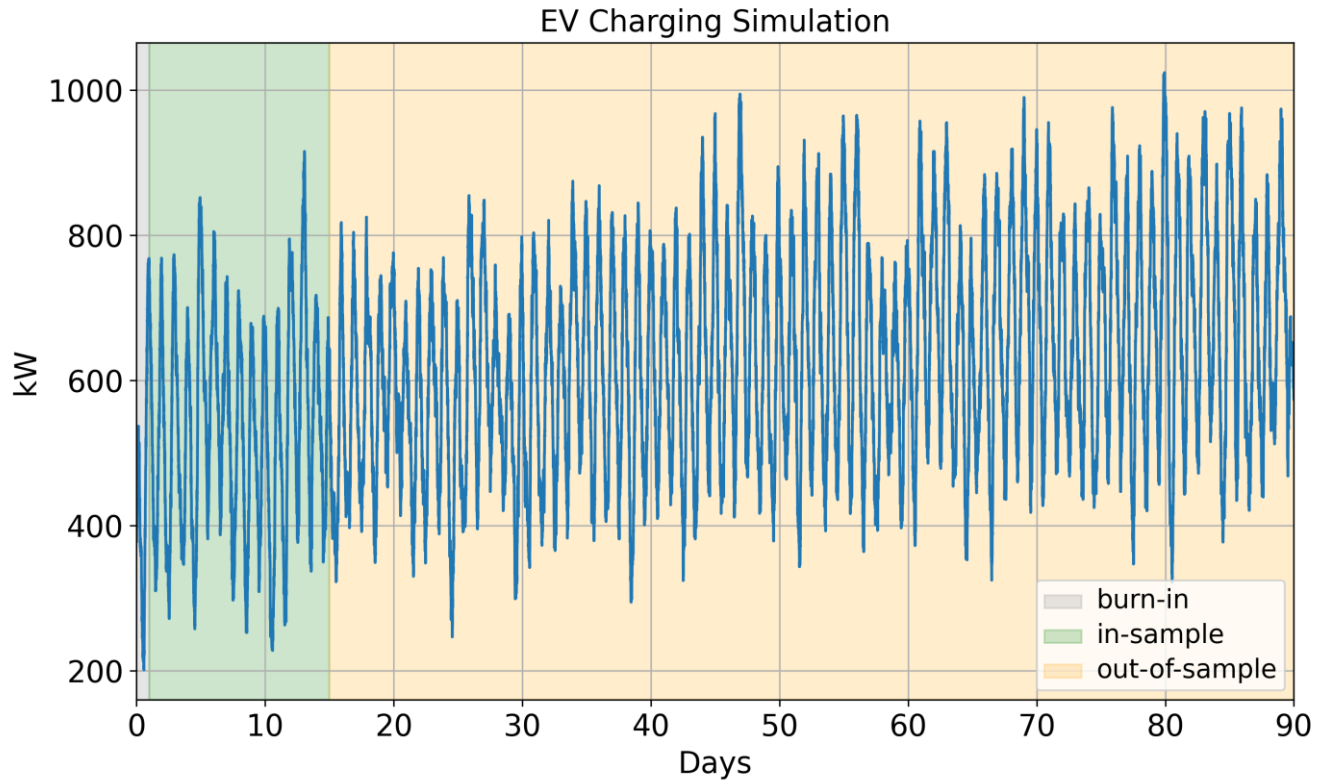
$$\begin{aligned}
 Z_{\text{JCC}}^* &= \min_{\mathbf{q}, \mathbf{s}, t, \mathbf{x}} \mathbf{c}^\top \mathbf{x} \\
 \text{s.t.} \quad & \varepsilon N t - \mathbf{e}^\top \mathbf{s} \geq \theta N \\
 & \frac{\mathbf{b}_m^\top \hat{\boldsymbol{\xi}}_i + b_m - \mathbf{a}_m^\top \mathbf{x}}{\|\mathbf{b}_m\|_*} + M q_i \geq t - s_i \quad \forall m \in [M], i \in [N] \\
 & M(1 - q_i) \geq t - s_i \quad \forall i \in [N] \\
 & \mathbf{q} \in \{0, 1\}^N, \mathbf{s} \geq \mathbf{0}, \mathbf{x} \in \mathcal{X},
 \end{aligned}$$

where M is a suitably large (but finite) positive constant.

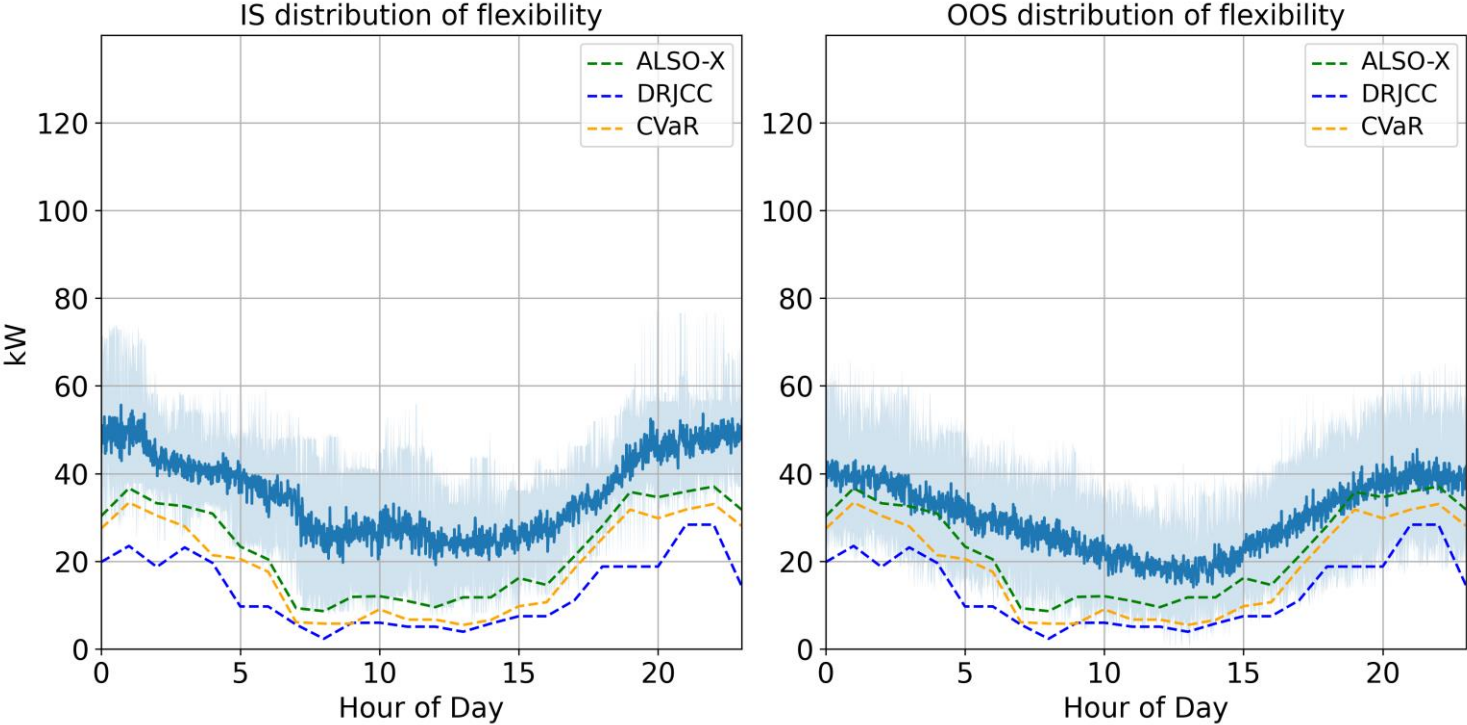
This results in a mixed-integer conic (or linear, depending on the norm) program.

[5] Z. Chen, D. Kuhn, and W. Wiesemann, “Data-driven chance constrained programs over Wasserstein balls,” *Operations Research*, vol. 72, no. 1, pp. 410–424, 2024

Input data: In-sample vs out-of-sample

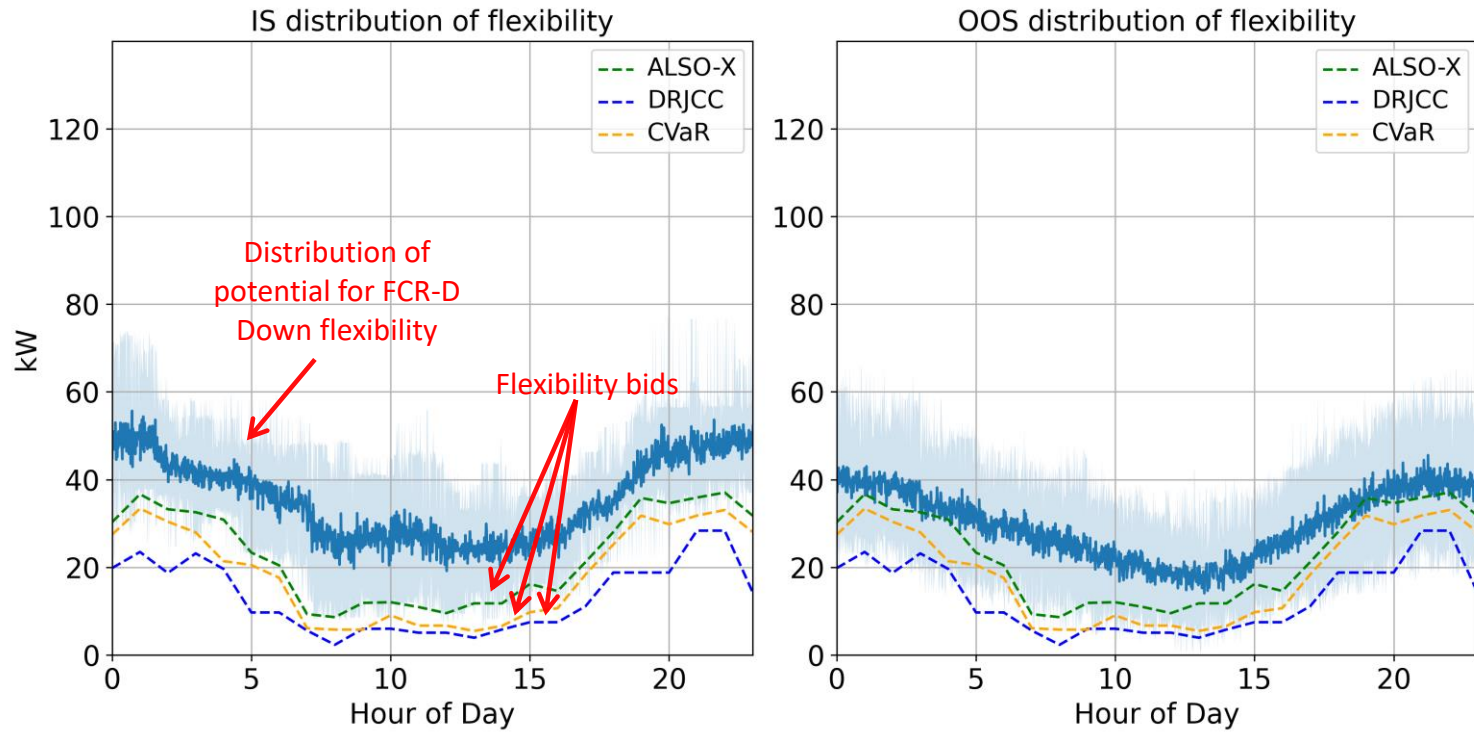


Results



IS: in-sample
 OOS: out-of-sample

Results



IS: in-sample
 OOS: out-of-sample

Some takeaways and potential future directions

- TSO requirements (both P90 and LER) can be modeled as a joint chance-constrained program.
- ALSO-X provides a good approximation of the chance constraint.
- CVaR is a conservative approach for solving a joint chance-constrained program.
- There is a synergy effect with more charging boxes in a bundle.

Potential future directions:

- Forecasting the baseline instead of using historical data for sampling (will it be useful?)
- Higher resolution data (enforcing constraints, e.g., per second, instead of minutes)
- Multi-market bidding (FCR-D, FCR-N, aFRR, FFR, etc)
- Does location of assets matter in low-inertia grids for frequency services?
- More heterogenous aggregation of stochastic assets (EVs + heat pumps +)

Further reading

G. Lunde, E. Damm, P. A. V. Gade, and JK, “Aggregator of electric vehicles bidding in Nordic FCR-D markets: A chance-constrained program,” <https://arxiv.org/abs/2404.12818>

P. A. V. Gade, H. Bindner, and JK, “Leveraging P90 requirement: Flexible resources bidding in Nordic ancillary service markets,” <https://arxiv.org/abs/2404.12807>

Thank you!



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