Exercise 1

Six examples are available in the papers on your table. Please check them in the next 15 minutes, and identify whether they are decomposable optimization problems or not. If so,

- how? Identify complicating variables/constraints!
- Number of complicating variables/constraints?
- Number of subproblems?

Then, check your results with your peer.

Exercise 1a

$$\underset{x_1,x_2,x_3,x_4,x_5,x_6}{\text{Maximize}} \; 4x_1 + 3x_2 + 2x_3 + 3x_4 + 2x_5 + 3x_6$$

subject to

$$x_1 - 2x_2 + 2x_6 \leq 3$$

$$2x_1 + x_2 + x_6 \leq 3$$

$$-2x_1 + 3x_2 + x_6 \leq 7$$

$$x_3 + 3x_6 \leq 4$$

$$2x_3 - x_6 \leq 3$$

$$x_4 \leq 1$$

$$2x_4 - 4x_5 + 3x_6 \leq 5$$

$$3x_4 + x_5 - x_6 \leq 4$$

Exercise 1b

Single-node single-hour market-clearing problem (total cost minimization) with 3 conventional generators and a single inelastic load:

$$\underset{g_1,g_2,g_3}{\text{Minimize}} \quad 10g_1 + 25g_2 + 30g_3$$

subject to

$$0 \leq g_1 \leq 100$$

$$0 \leq g_2 \leq 150$$

$$0 \leq g_3 \leq 200$$

$$g_1 + g_2 + g_3 = 350$$

Exercise 1c

Single-node single-hour market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load:

$$\underset{g_1, g_2, g_3, d_1}{\text{Maximize}}\ 40d_1 - 10g_1 - 25g_2 - 30g_3$$

subject to

$$0 \le g_1 \le 100$$

$$0 \le g_2 \le 150$$

$$0 \le g_3 \le 200$$

$$0 \le d_1 \le 350$$

$$g_1 + g_2 + g_3 = d_1$$

Exercise 1d

Single-node multi-hour (index: *h*) market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load:

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \quad \sum_{h}[40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Exercise 1e

Single-node multi-hour (index: *h*) market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load (including ramping constraints):

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h} [40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$-25 \leq [g_{h,1} - g_{(h-1),1}] \leq 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Exercise 1f

Single-node single-year (static) generation expansion problem, considering two existing generators (units 1 and 2) and one candidate generator (unit 3)

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad 15000x_3 + \sum_h [10g_{h,1} + 25g_{h,2} + 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \geq 0$$

Exercise 2: Benders' Decomposition

Write the formulation of Master and Subproblems (in a compact way) for the following two-stage stochastic optimal power flow (OPF) problem. The two stages are day-ahead (DA) and real-time (RT).

$$\underset{p_g^{\mathrm{DA}},\ p_{g,\omega}^{\mathrm{RT}},\ p_{d,\omega}^{\mathrm{shed}}}{\text{Minimize}} \quad \mathbf{Cost}^{\mathrm{DA}}(p_g^{\mathrm{DA}}) + \mathbb{E}_\omega\left[\mathbf{Cost}^{\mathrm{RT}}(p_{g,\omega}^{\mathrm{RT}},\ p_{d,\omega}^{\mathrm{shed}})\right]$$

subject to:

$$\mathbf{f}(p_g^{\mathrm{DA}}) \leq 0$$

$$\mathbf{g}(p_{g,\omega}^{\mathrm{RT}},\ p_{d,\omega}^{\mathrm{shed}}) \leq 0 \qquad \forall\omega$$

$$\mathbf{h}(p_g^{\mathrm{DA}},\ p_{g,\omega}^{\mathrm{RT}},\ p_{d,\omega}^{\mathrm{shed}}) \leq 0 \qquad \forall\omega$$

Exercise 3: ADMM

Consider a two-stage stochastic programming problem, e.g., a two-settlement market-clearing problem with day ahead (DA) and real time (RT) stages:

$$\underset{p_g^{\mathrm{DA}},\ p_{g,\omega}^{\mathrm{RT}},\ p_{d,\omega}^{\mathrm{shed}}}{\text{Minimize}} \quad \mathbf{Cost}^{\mathrm{DA}}(p_g^{\mathrm{DA}}) + \mathbb{E}_\omega \left[\mathbf{Cost}^{\mathrm{RT}}(p_{g,\omega}^{\mathrm{RT}},\ p_{d,\omega}^{\mathrm{shed}})\right]$$

subject to:

$$\mathbf{f}(p_g^{\mathrm{DA}}) \leq 0$$

$$\mathbf{g}(p_{g,\omega}^{\mathrm{RT}},\ p_{d,\omega}^{\mathrm{shed}}) \leq 0 \qquad \forall \omega$$

$$\mathbf{h}(p_g^{\mathrm{DA}},\ p_{g,\omega}^{\mathrm{RT}},\ p_{d,\omega}^{\mathrm{shed}}) \leq 0 \qquad \forall \omega$$

Can this problem be solved by "consensus ADMM"? If so, how? Write the formulation.

Guide: Think of relaxing the non-anticipativity conditions, and to have one subproblem per scenario.