DTU CEE Summer School 2019

Data-Driven Analytics and Optimization for Energy Systems

June 16-21 2019

# Distributed Optimization

## Lecture 1: Optimization problems with decomposable structure

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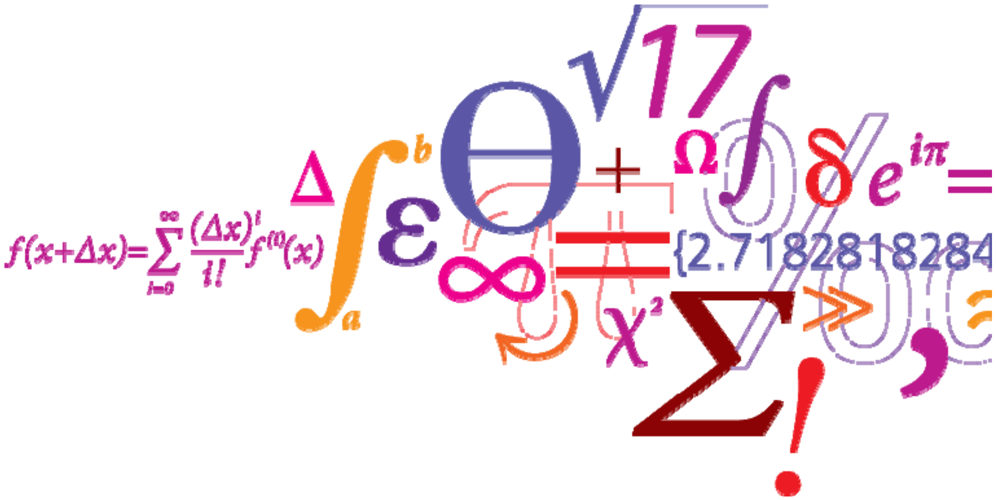


**June 21, 2019**

DTU Electrical Engineering
Department of Electrical Engineering

# Agenda

| Lecture | Topic | Time period |
|---|---|---|
| 1 | Optimization problems with decomposable structures (and Exercise 1 including discussion) | 9:15 – 10:00 |
| | Break | 10:00 – 10:10 |
| 2 | Benders’ decomposition (and Exercise 2) | 10:10 – 11:00 |
| | Break | 11:00 – 10:10 |
| | Discussion on Exercise 2 | 11:10 – 11:30 |
| 3 | Part 1: Lagrangian relaxation | 11:30 – 12:00 |
| | Lunch | 12:00 – 13:30 |
| | Side Talk (Introduction to MOSEK by M. Adamaszek) | 13:30 – 14:00 |
| 3 | Part 2: Augmented Lagrangian relaxation | 14:00 – 14:30 |
| | Break | 14:30 – 14:40 |
| 4 | Exchange and consensus ADMM (and Exercise 3) | 14:40 – 15:30 |

# Learning objectives

After Lecture 1, you are expected to be able to:

- Explain the need for decomposition
- Identify whether each optimization problem is decomposable or not (if so, how?)

# Decomposition: main idea

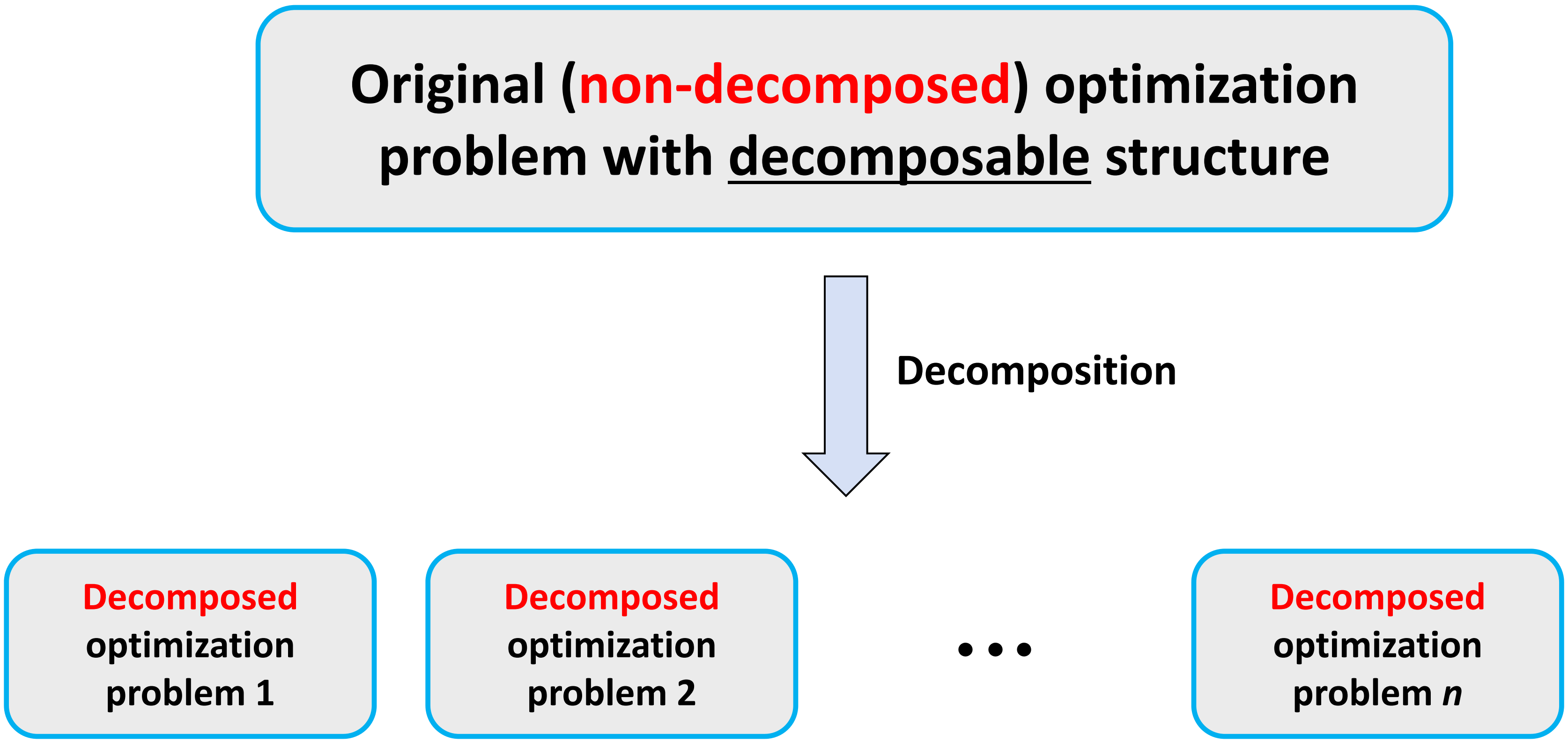
**Original (non-decomposed) optimization problem with decomposable structure**

# Decomposition: main idea

**Original (non-decomposed) optimization problem with decomposable structure**

↓ **Decomposition**

# Decomposition: main idea




Each decomposed problem is easier to solve than the original (non-decomposed) problem!

# Decomposition: motivation

- Why do we need decomposition in energy systems?

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  - Planning problems (e.g., investment)

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  - Operational problems (e.g., unit commitment)
    - Need to be computationally tractable
    - Need to be solved in a specific time period
  - Planning problems (e.g., investment)
    - Need to be computationally tractable

# Decomposition: motivation

- Why do we need decomposition in energy systems?
  - Operational problems (e.g., unit commitment)
    - Need to be computationally tractable
    - Need to be solved in a specific time period
  - Planning problems (e.g., investment)
    - Need to be computationally tractable

Any other reason for decomposition?

# Decomposable structure

Optimization problems with decomposable structure:

- Problems with complicating variable(s):

- Problems with complicating constraint(s):

# Decomposable structure

Optimization problems with decomposable structure:

- Problems with complicating variable(s):

  The original problem is decomposed if the complicating variables are fixed to given values!

- Problems with complicating constraint(s):

  The original problem is decomposed if the complicating constraints are relaxed (removed)!

# Decomposable structure

Optimization problems with decomposable structure:

- Problems with complicating variable(s):

  The original problem is decomposed if the complicating variables are fixed to given values!

- Problems with complicating constraint(s):

  The original problem is decomposed if the complicating constraints are relaxed (removed)!

In the literature, “complicating” variables/constraints are also called as “coupling” variables/constraint!

# Features of decomposition techniques

- Iterative solution techniques

- Original optimization problem decomposes to:
  - A single master problem (not necessarily an optimization problem)
  - A set of subproblems

# Decomposable structures

Optimization problems with complicating constraint(s)

Example: a linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,x_3,\\ y_1,y_2,\\ z_1,z_2,z_3}}{\text{Minimize}} \quad A_1x_1 + A_2x_2 + A_3x_3 + B_1y_1 + B_2y_2 + C_1z_1 + C_2z_2 + C_3z_3$$

Subject to

$$E_{11}x_1 + E_{12}x_2 + E_{13}x_3 \geq F_1$$

$$E_{21}x_1 + E_{22}x_2 + E_{23}x_3 \geq F_2$$

$$E_{31}y_1 + E_{32}y_2 \geq F_3$$

$$E_{41}z_1 + E_{42}z_2 + E_{43}z_3 \geq F_4$$

$$E_{51}z_1 + E_{52}z_2 + E_{53}z_3 \geq F_5$$

$$E_{61}x_1 + E_{62}x_2 + E_{63}x_3 + E_{64}y_1 + E_{65}y_2 + E_{66}z_1 + E_{67}z_2 + E_{68}z_3 \geq F_6$$

# Decomposable structures

## Optimization problems with complicating constraint(s)

Example: a linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,x_3,\\ y_1,y_2,\\ z_1,z_2,z_3}}{\text{Minimize}} \quad A_1 x_1 + A_2 x_2 + A_3 x_3 + B_1 y_1 + B_2 y_2 + C_1 z_1 + C_2 z_2 + C_3 z_3$$

Subject to

$$E_{11} x_1 + E_{12} x_2 + E_{13} x_3 \geq F_1$$

$$E_{21} x_1 + E_{22} x_2 + E_{23} x_3 \geq F_2$$

$$E_{31} y_1 + E_{32} y_2 \geq F_3$$

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# Decomposable structures

Optimization problems with complicating constraint(s)

Example: a linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,x_3,\\ y_1,y_2,\\ z_1,z_2,z_3}}{\text{Minimize}} \quad A_1 x_1 + A_2 x_2 + A_3 x_3 + B_1 y_1 + B_2 y_2 + C_1 z_1 + C_2 z_2 + C_3 z_3$$

Subject to Constraints including only variables $x$

$$E_{11} x_1 + E_{12} x_2 + E_{13} x_3 \geq F_1$$

$$E_{21} x_1 + E_{22} x_2 + E_{23} x_3 \geq F_2$$

Constraint including only variables $y$

$$E_{31} y_1 + E_{32} y_2 \geq F_3$$

Constraints including only variables $z$

$$E_{41} z_1 + E_{42} z_2 + E_{43} z_3 \geq F_4$$

$$E_{51} z_1 + E_{52} z_2 + E_{53} z_3 \geq F_5$$

$$E_{61} x_1 + E_{62} x_2 + E_{63} x_3 + E_{64} y_1 + E_{65} y_2 + E_{66} z_1 + E_{67} z_2 + E_{68} z_3 \geq F_6$$

# Decomposable structures

## Optimization problems with complicating constraint(s)

Example: a linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,x_3,\\ y_1,y_2,\\ z_1,z_2,z_3}}{\text{Minimize}} \quad A_1 x_1 + A_2 x_2 + A_3 x_3 + B_1 y_1 + B_2 y_2 + C_1 z_1 + C_2 z_2 + C_3 z_3$$

Subject to Constraints including only variables $x$

$$E_{11} x_1 + E_{12} x_2 + E_{13} x_3 \geq F_1$$

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Constraint including only variables $y$

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Constraints including only variables $z$

$$E_{41} z_1 + E_{42} z_2 + E_{43} z_3 \geq F_4$$

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$$E_{61} x_1 + E_{62} x_2 + E_{63} x_3 + E_{64} y_1 + E_{65} y_2 + E_{66} z_1 + E_{67} z_2 + E_{68} z_3 \geq F_6$$

This is a complicating constraint: If relaxed (removed), then the original problem decomposes to three smaller optimization problems (subproblems)!

# Decomposable structures

Optimization problems with complicating constraint(s)

Example: a linear programming (LP) as the original problem

**Subproblem 1:**

$$\underset{x_1, x_2, x_3}{\text{Minimize}} \quad A_1 x_1 + A_2 x_2 + A_3 x_3$$

Subject to

Constraints including only variables $x$

$$E_{11} x_1 + E_{12} x_2 + E_{13} x_3 \geq F_1$$

$$E_{21} x_1 + E_{22} x_2 + E_{23} x_3 \geq F_2$$

# Decomposable structures

Optimization problems with complicating constraint(s)

Example: a linear programming (LP) as the original problem

**Subproblem 2:**

$$\underset{y_1, y_2}{\text{Minimize}} \quad B_1 y_1 + B_2 y_2$$

Subject to

Constraint including only variables $y$

$$E_{31} y_1 + E_{32} y_2 \geq F_3$$

# Decomposable structures

Optimization problems with complicating constraint(s)

Example: a linear programming (LP) as the original problem

**Subproblem 3:**

$$\underset{z_1, z_2, z_3}{\text{Minimize}} \quad C_1 z_1 + C_2 z_2 + C_3 z_3$$

Subject to

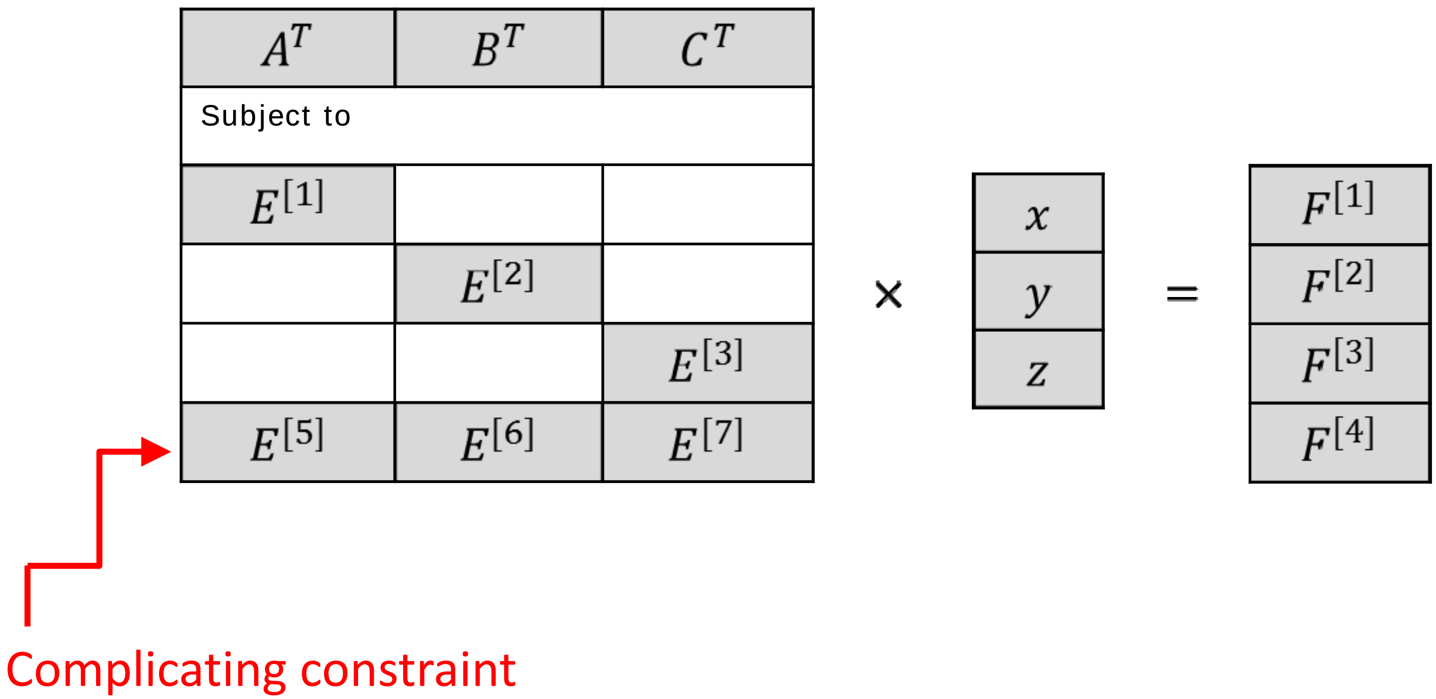
Constraints including only variables $z$

$$E_{41} z_1 + E_{42} z_2 + E_{43} z_3 \geq F_4$$

$$E_{51} z_1 + E_{52} z_2 + E_{53} z_3 \geq F_5$$

# Decomposable structures

Optimization problems with complicating constraint(s)



# Decomposable structures

Optimization problems with complicating variable(s)

Example: a linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,\\ y_1,y_2,\\ z_1,z_2,\\ \beta}}{\text{Minimize}} \quad A_1x_1 + A_2x_2 + B_1y_1 + B_2y_2 + C_1z_1 + C_2z_2 + D_1\,\beta$$

Subject to

$$E_{11}x_1 + E_{12}\,x_2 + E_{13}\,\beta \geq F_1$$
$$E_{21}x_1 + E_{22}\,x_2 + E_{23}\beta \geq F_2$$

$$E_{31}y_1 + E_{32}\,y_2 + E_{33}\,\beta \geq F_3$$

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# Decomposable structures

## Optimization problems with complicating variable(s)

Example: a linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,\\ y_1,y_2,\\ z_1,z_2,\\ \beta}}{\text{Minimize}} \quad A_1x_1 + A_2x_2 + B_1y_1 + B_2y_2 + C_1z_1 + C_2z_2 + D_1\beta$$

Subject to

$$E_{11}x_1 + E_{12}x_2 + E_{13}\beta \geq F_1$$
$$E_{21}x_1 + E_{22}x_2 + E_{23}\beta \geq F_2$$

$$E_{31}y_1 + E_{32}y_2 + E_{33}\beta \geq F_3$$

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# Decomposable structures

Optimization problems with complicating variable(s)

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$$\underset{\substack{x_1,x_2,\\ y_1,y_2,\\ z_1,z_2,\\ \beta}}{\text{Minimize}} \quad A_1 x_1 + A_2 x_2 + B_1 y_1 + B_2 y_2 + C_1 z_1 + C_2 z_2 + D_1 \beta$$

Subject to

$$E_{11} x_1 + E_{12} x_2 + E_{13} \beta \geq F_1$$
$$E_{21} x_1 + E_{22} x_2 + E_{23} \beta \geq F_2$$

$$E_{31} y_1 + E_{32} y_2 + E_{33} \beta \geq F_3$$

$$E_{41} z_1 + E_{42} z_2 + E_{43} \beta \geq F_4$$
$$E_{51} z_1 + E_{52} z_2 + E_{53} \beta \geq F_5$$

**$\beta$ is a complicating variable, i.e., if it is fixed to a given value ($\beta^{\text{fixed}}$), then the original problem decomposes to 3 smaller optimization problems (subproblems)!**

# Decomposable structures

## Optimization problems with complicating variable(s)

Example: a linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,\\ y_1,y_2,\\ z_1,z_2,\\ \beta}}{\text{Minimize}} \quad A_1x_1 + A_2x_2 + B_1y_1 + B_2y_2 + C_1z_1 + C_2z_2 + \underbrace{D_1\,\beta}$$

Fixed term, it can be removed from objective function

Subject to

Constraints including only variables $x$

$$E_{11}x_1 + E_{12}\,x_2 + E_{13}\,\beta \geq F_1$$
$$E_{21}x_1 + E_{22}\,x_2 + E_{23}\beta \geq F_2$$

$$E_{31}y_1 + E_{32}\,y_2 + E_{33}\,\beta \geq F_3$$

$$E_{41}z_1 + E_{42}\,z_2 + E_{43}\,\beta \geq F_4$$
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# Decomposable structures

Optimization problems with complicating variable(s)

Example: a linear programming (LP) as the original problem

**Subproblem 1:**

$$\underset{x_1, x_2}{\text{Minimize}} \quad A_1 x_1 + A_2 x_2$$

Subject to

Constraints including only variables $x$

$$E_{11} x_1 + E_{12} x_2 \geq F_1 - E_{13}\, \beta^{\text{fixed}}$$
$$E_{21} x_1 + E_{22} x_2 \geq F_2 - E_{23}\, \beta^{\text{fixed}}$$

# Decomposable structures

Optimization problems with complicating variable(s)

Example: a linear programming (LP) as the original problem

**Subproblem 2:**

$$\underset{y_1,y_2}{\text{Minimize}}\, B_1 y_1 + B_2 y_2$$

Subject to

Constraint including only variables $y$

$$E_{31} y_1 + E_{32}\, y_2 \geq F_3 \; - E_{33}\, \beta^{\text{fixed}}$$

# Decomposable structures

Optimization problems with complicating variable(s)

Example: a linear programming (LP) as the original problem

**Subproblem 3:**

$$\underset{z_1, z_2}{\text{Minimize}}\; C_1 z_1 + C_2 z_2$$

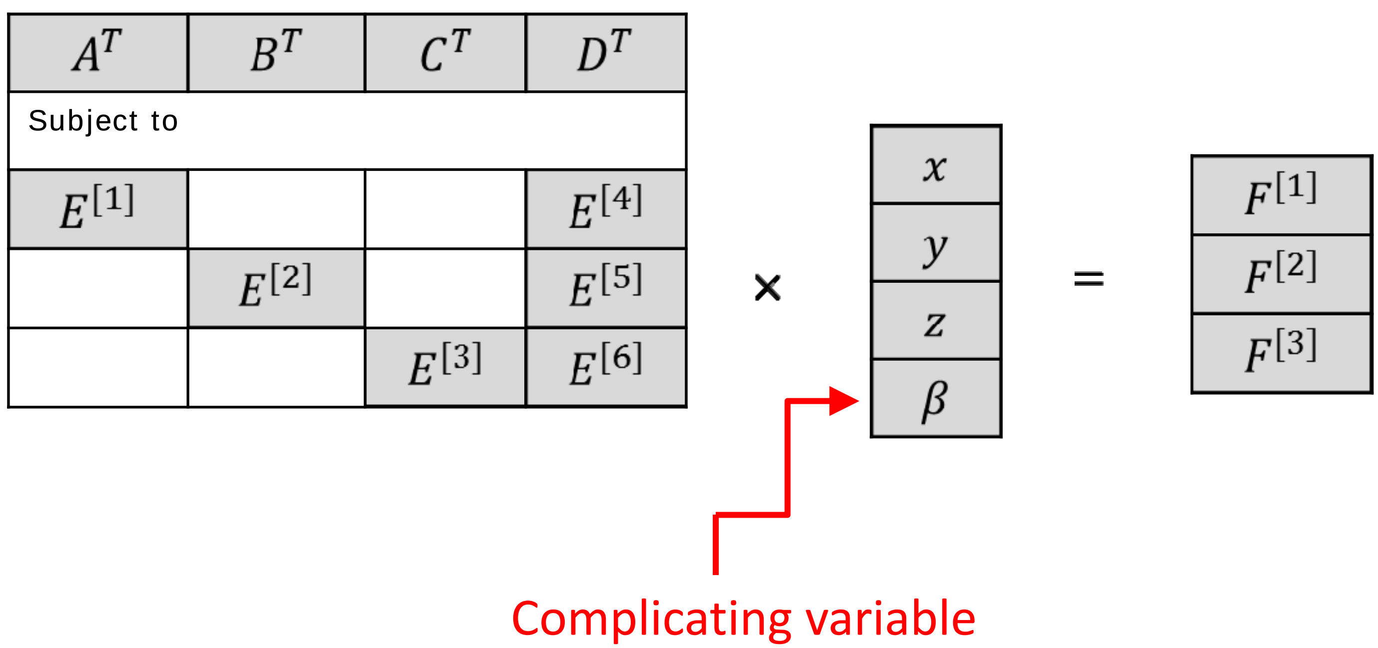
Subject to

Constraints including only variables $z$

$$E_{41} z_1 + E_{42} z_2 \geq F_4 - E_{43}\, \beta^{\text{fixed}}$$
$$E_{51} z_1 + E_{52} z_2 \geq F_5 - E_{53}\, \beta^{\text{fixed}}$$

# Decomposable structures

Optimization problems with complicating variable(s)



# Exercise 1

Six examples are available in the papers on your table. Please check them in the next 15 minutes, and identify whether they are decomposable optimization problems or not. If so,

- how? Identify complicating variables/constraints!
- Number of complicating variables/constraints?
- Number of subproblems?

Then, check your results with your peer.

# Example 1

$$\underset{x_1,x_2,x_3,x_4,x_5,x_6}{\text{Maximize}} \; 4x_1 + 3x_2 + 2x_3 + 3x_4 + 2x_5 + 3x_6$$

subject to

$$x_1 - 2x_2 + 2x_6 \leq 3$$

$$2x_1 + x_2 + x_6 \leq 3$$

$$-2x_1 + 3x_2 + x_6 \leq 7$$

$$x_3 + 3x_6 \leq 4$$

$$2x_3 - x_6 \leq 3$$

$$x_4 \leq 1$$

$$2x_4 - 4x_5 + 3x_6 \leq 5$$

$$3x_4 + x_5 - x_6 \leq 4$$

# Example 1

$$\underset{x_1,x_2,x_3,x_4,x_5,x_6}{\text{Maximize}} \; 4x_1 + 3x_2 + 2x_3 + 3x_4 + 2x_5 + 3x_6$$

subject to

$$x_1 - 2x_2 + 2x_6 \leq 3$$
$$2x_1 + x_2 + x_6 \leq 3$$
$$-2x_1 + 3x_2 + x_6 \leq 7$$
$$x_3 + 3x_6 \leq 4$$
$$2x_3 - x_6 \leq 3$$
$$x_4 \leq 1$$
$$2x_4 - 4x_5 + 3x_6 \leq 5$$
$$3x_4 + x_5 - x_6 \leq 4$$

**$x_6$ is a complicating variable:**
if fixed to a given value, then the original problem decomposes to 3 subproblems

# Example 2

**Single-node single-hour market-clearing problem (total cost minimization) with 3 conventional generators and a single inelastic load:**

$$\underset{g_1,g_2,g_3}{\text{Minimize}} \quad 10g_1 + 25g_2 + 30g_3$$

subject to

$$0 \leq g_1 \leq 100$$

$$0 \leq g_2 \leq 150$$

$$0 \leq g_3 \leq 200$$

$$g_1 + g_2 + g_3 = 350$$

# Example 2

**Single-node single-hour market-clearing problem (total cost minimization) with 3 conventional generators and a single inelastic load:**

$$\underset{g_1,g_2,g_3}{\text{Minimize}} \quad 10g_1 + 25g_2 + 30g_3$$

subject to

$$0 \le g_1 \le 100$$

$$0 \le g_2 \le 150$$

$$0 \le g_3 \le 200$$

$$g_1 + g_2 + g_3 = 350$$

→ **Complicating constraint:** if relaxed, then the original problem decomposes to 3 subproblems, <u>one per generator</u>

# Example 3

**Single-node single-hour market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load:**

$$\underset{g_1,g_2,g_3,d_1}{\text{Maximize}}\ 40d_1 - 10g_1 - 25g_2 - 30g_3$$

subject to

$$0 \leq g_1 \leq 100$$

$$0 \leq g_2 \leq 150$$

$$0 \leq g_3 \leq 200$$

$$0 \leq d_1 \leq 350$$

$$g_1 + g_2 + g_3 = d_1$$

# Example 3

**Single-node single-hour market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load:**

$$\underset{g_1,g_2,g_3,d_1}{\text{Maximize}}\ 40d_1 - 10g_1 - 25g_2 - 30g_3$$

subject to

$$0 \le g_1 \le 100$$

$$0 \le g_2 \le 150$$

$$0 \le g_3 \le 200$$

$$0 \le d_1 \le 350$$

$$g_1 + g_2 + g_3 = d_1$$

**Complicating constraint:**
if relaxed, then the original problem decomposes to 4 subproblems, one per agent (i.e., 3 generators and 1 demand)

# Example 4

**Single-node multi-hour (index: *h*) market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load:**

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h}\left[40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}\right]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

# Example 4

**Single-node multi-hour (index: *h*) market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load:**

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h} [40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

**Complicating constraints:**
if relaxed, then the original problem decomposes to a set of subproblems, one per agent per hour

# Example 4

Single-node multi-hour (index: *h*) market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load:

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h} [40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

- Number of complicating constraints: ?
- Number of subproblem: ?

# Example 4

**Single-node multi-hour (index: *h*) market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load:**

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h} [40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$\boxed{g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \qquad \forall h}$$

- **Number of complicating constraints: |h|**
- **Number of subproblem: 4|h|**

# Example 5

Single-node multi-hour (index: *h*) market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load (including ramping constraints):

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \quad \sum_{h} \left[40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}\right]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$-25 \leq \left[g_{h,1} - g_{(h-1),1}\right] \leq 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

# Example 5

**Single-node multi-hour (index: *h*) market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load (including ramping constraints):**

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h} \left[40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}\right]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$-25 \leq \left[g_{h,1} - g_{(h-1),1}\right] \leq 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Investigate the following three options:

1) relax ramping constraints only,
2) relax balance constraints only,
3) relax both.

# Example 5

**Single-node multi-hour (index: *h*) market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load (including ramping constraints):**

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h}\left[40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}\right]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$-25 \leq \left[g_{h,1} - g_{(h-1),1}\right] \leq 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Option 1:

- Number of complicating constraints: ?
- Number of subproblem: ?

# Example 5

**Single-node multi-hour (index: *h*) market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load (including ramping constraints):**

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h} [40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$-25 \leq [g_{h,1} - g_{(h-1),1}] \leq 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Option 1:

- Number of complicating constraints: |h|
- Number of subproblem: |h|

# Example 5

**Single-node multi-hour (index: *h*) market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load (including ramping constraints):**

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h} \left[40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}\right]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$-25 \leq \left[g_{h,1} - g_{(h-1),1}\right] \leq 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Option 2:

- Number of complicating constraints:?
- Number of subproblem: ?

# Example 5

**Single-node multi-hour (index: *h*) market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load (including ramping constraints):**

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h} [40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$-25 \leq [g_{h,1} - g_{(h-1),1}] \leq 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Option 2:

- Number of complicating constraints: |h|
- Number of subproblem: 3|h|+1

# Example 5

**Single-node multi-hour (index: *h*) market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load (including ramping constraints):**

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h} \left[40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}\right]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$-25 \leq \left[g_{h,1} - g_{(h-1),1}\right] \leq 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Option 3:

- Number of complicating constraints: ?
- Number of subproblem: ?

# Example 5

**Single-node multi-hour (index: *h*) market-clearing problem (social welfare maximization) with 3 conventional generators and a single elastic load (including ramping constraints):**

$$\underset{g_{h,1}, g_{h,2}, g_{h,3}, d_{h,1}}{\text{Maximize}} \sum_{h} [40 d_{h,1} - 10 g_{h,1} - 25 g_{h,2} - 30 g_{h,3}]$$

subject to

$$0 \le g_{h,1} \le 100 \quad \forall h$$

$$0 \le g_{h,2} \le 150 \quad \forall h$$

$$0 \le g_{h,3} \le 200 \quad \forall h$$

$$0 \le d_{h,1} \le D_{h,1} \quad \forall h$$

$$-25 \le [g_{h,1} - g_{(h-1),1}] \le 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Option 3:

- Number of complicating constraints: 2|h|
- Number of subproblem: 4|h|

# Example 6

Single-node single-year (static) generation expansion problem, considering two existing generators (units 1 and 2) and one candidate generator (unit 3)

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad 15000x_3 + \sum_h [10g_{h,1} + 25g_{h,2} + 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \geq 0$$

# Example 6

**Single-node single-year (static) generation expansion problem, considering two existing generators (units 1 and 2) and one candidate generator (unit 3)**

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad \underbrace{15000x_3}_{\text{Expansion cost of candidate unit}} + \underbrace{\sum_h [10g_{h,1} + 25g_{h,2} + 30g_{h,3}]}_{\text{Operational cost of (existing and candidate) units}}$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \geq 0$$

# Example 6

**Single-node single-year (static) generation expansion problem, considering two existing generators (units 1 and 2) and one candidate generator (unit 3)**

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad 15000x_3 + \sum_h [10g_{h,1} + 25g_{h,2} + 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \geq 0$$

Investigate the following two options:

1) $x_3$ is a complicating **variable**,
2) Balance conditions are complicating **constraints**.

# Example 6

**Single-node single-year (static) generation expansion problem, considering two existing generators (units 1 and 2) and one candidate generator (unit 3)**

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad 15000x_3 + \sum_h [10g_{h,1} + 25g_{h,2} + 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \geq 0$$

Option 1 (fixing $x_3$):

- Number of complicating variables: ?
- Number of subproblem: ?

# Example 6

**Single-node single-year (static) generation expansion problem, considering two existing generators (units 1 and 2) and one candidate generator (unit 3)**

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad 15000x_3 + \sum_h [10g_{h,1} + 25g_{h,2} + 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \geq 0$$

Option 1 (fixing $x_3$):

- Number of complicating variables: 1
- Number of subproblem: |h|

# Example 6

**Single-node single-year (static) generation expansion problem, considering two existing generators (units 1 and 2) and one candidate generator (unit 3)**

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad 15000x_3 + \sum_h [10g_{h,1} + 25g_{h,2} + 30g_{h,3}]$$

subject to

$$0 \le g_{h,1} \le 100 \quad \forall h$$

$$0 \le g_{h,2} \le 150 \quad \forall h$$

$$0 \le g_{h,3} \le x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \ge 0$$

Option 2 (relaxing balance constraints):

- Number of complicating constraints: ?
- Number of subproblem: ?

# Example 6

**Single-node single-year (static) generation expansion problem, considering two existing generators (units 1 and 2) and one candidate generator (unit 3)**

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad 15000x_3 + \sum_h [10g_{h,1} + 25g_{h,2} + 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \geq 0$$

Option 2 (relaxing balance constraints):

- Number of complicating constraints: |h|
- Number of subproblem: 2|h|+1

# Optional Assignment 1

Consider the following generation expansion problem, which is decomposable by fixing complicating variable $x_3$. Derive its corresponding dual optimization, and check whether it is decomposable too. Any general conclusion?

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad 15000x_3 + \sum_h \left[10g_{h,1} + 25g_{h,2} + 30g_{h,3}\right]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \geq 0$$

# Optional Assignment 2

Consider the following compact form of stochastic market-clearing problem. List all options for decomposing this problem, and indicate the complicating variables/constraints. The ideal options decompose the original problem by scenario (i.e., one subproblem per scenario), since this problem may include a large number of scenarios.

$$\underset{p_g^{\mathrm{DA}},\ p_{g,\omega}^{\mathrm{RT}},\ p_{d,\omega}^{\mathrm{shed}}}{\text{Minimize}} \quad \mathbf{Cost}^{\mathrm{DA}}(p_g^{\mathrm{DA}}) + \mathbb{E}_\omega \left[\mathbf{Cost}^{\mathrm{RT}}(p_{g,\omega}^{\mathrm{RT}},\ p_{d,\omega}^{\mathrm{shed}})\right]$$

subject to:

$$\mathbf{f}(p_g^{\mathrm{DA}}) \leq 0$$

$$\mathbf{g}(p_{g,\omega}^{\mathrm{RT}},\ p_{d,\omega}^{\mathrm{shed}}) \leq 0 \qquad \forall\omega$$

$$\mathbf{h}(p_g^{\mathrm{DA}},\ p_{g,\omega}^{\mathrm{RT}},\ p_{d,\omega}^{\mathrm{shed}}) \leq 0 \qquad \forall\omega$$

# Optional Assignment 3

Think of any other optimization problem in energy systems – it could be your current or previous (or future!) research work. Is it decomposable? Write it in a compact form (similar to optional assignment 2) and then discuss how it is decomposable.

# Thanks for your attention!

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