Distributionally Robust Energy and Reserve Dispatch with Wasserstein Metric

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1 Introduction

In this project, the energy and reserve dispatch problem under the uncertainty of renewable energy production is solved. The focus is on wind power production that is modeled by $W(\mu + \xi)$, where $\mu \in \mathbb{R}^Z$ is the mean power production, $\xi \in \mathbb{R}^Z$ is a random variable with zero mean and $W \in \mathbb{R}^{Z \times Z}$ a diagonal matrix that contains the capacity of the wind farms. By utilizing such a representation of uncertainty, the recourse actions can be restricted to linear decision rules that results in modeling power production from conventional generators by $p + Y\xi$, where $p \in \mathbb{R}^G_+$ is the day-ahead dispatch of power plants and $Y\xi$ the real-time adjustment for $Y \in \mathbb{R}^{G \times Z}$. The electricity demand is assumed perfectly known in this project.

The objectives to be achieved at the end of the assignment are the following:

- Understand the concepts of stochastic programming and distributionally robust optimization
- Formulate the energy and reserves problem in a distributionally robust framework with the ambiguity set defined by the Wasserstein metric

- Provide a tractable reformulation for the objective function and individual chance constraints of the distributionally robust energy and reserve dispatch problem
- Implement the energy and reserve dispatch problem and perform an out-of-sample testing to evaluate the solutions obtained
- Analyze how the ambiguity size and the number of available data affects the solutions

2 System data

Consider a single-node system with five conventional generators, two wind farms and a single demand. Tables 1 and 2 provide the data for the conventional generators.

Table 1: Conventional generator technical data

Generator	Minimum power production ($p^{\min}$) [MW]	Maximum power production ($p^{\max}$) [MW]	Maximum reserve availability ($r^{\max}$) [MW]
G1	0	40	16
G2	0	170	68
G3	0	520	208
G4	0	200	80
G5	0	600	240

Table 2: Conventional generator cost data

Generator	Power production cost (c) [$/MWh]	Up reg. reserve cost ($\overline{c}$) [$/MW]	Down reg. reserve cost ($\underline{c}$) [$/MW]
G1	14	2.8	2.8
G2	15	3	3
G3	30	6	6
G4	40	8	8
G5	10	2	2

The capacity of each of the two wind farms is 200 MW and the total electricity demand is 1000 MW. Note that wind power production is offered at zero cost.

3 Tasks: Implement the Distributionally Robust Energy and Reserve Dispatch with the Wasserstein Metric

This project has five steps:

1. Based on the aforementioned description of power production for conventional and wind generators, formulate the optimization model for the energy and reserves dispatch in a stochastic programming (*SP*) framework. A description for co-optimization of energy and reserves dispatch problem can be found in [1, Section 3.2.3]. The goal is to minimize the total expected cost and the constraints that contain random variables would need to be formulated as individual chance constraints. What kind of information would one need in order to solve *SP* regarding the characterization of the uncertain parameter ξ? Additional useful information for the *SP* formulation can be found in [2, Section II.A] and in model *CC-OPF* of [3].

2. Reformulate the energy and reserves dispatch problem in a distributionally robust (*DR*) framework. Comment on the main differences between the *SP* and *DR* approaches in terms of problem formulation. What kind of information would one need in order to solve *DR* model regarding the characterization of the uncertain parameter ξ? Additional useful information for the *DR* formulation can be found in [3] and [4].

3. Consider an ambiguity set $\mathcal{P}$ defined as a Wasserstein ball centered at the empirical distribution $\widehat{\mathbb{P}}_n$ with a radius ρ as

$$\mathcal{P} := \left\{ \mathbb{Q} \in \mathcal{M}(\Xi) : W_q(\widehat{\mathbb{P}}_n, \mathbb{Q}) \leq \rho \right\}, \tag{1}$$

   where $\widehat{\mathbb{P}}_n$ is the discrete empirical distribution that contain N samples from historical data of wind power production and W_q is the type-1 Wasserstein metric.

   (a) Based on [5, Corollary 5.1] provide the reformulation of the worst-case expectation in the objective function of *DR* model. In the reformulation, you may omit the constraints of the uncertainty set given in [5, Corollary 5.1], i.e. $C\xi \leq d$.

(b) For a generic type of individual chance constraints, the feasible set of each constraint is defined as

$$\Omega := \left\{ x \in \mathbb{R}^N : \inf_{\mathbb{Q} \in \mathcal{P}} \mathbb{Q}(\alpha(x)^\top \xi + \beta(x) \leq 0) \geq 1 - \epsilon \right\}. \tag{2}$$

As demonstrated in [6, Section 2.1], the Conditional Value-at-Risk (CVaR) measure can be utilized to provide a conservative approximation for the distributionally robust chance constraint with the feasible set,

$$\Omega^C := \left\{ x \in \mathbb{R}^N : \sup_{\mathbb{Q} \in \mathcal{P}} \mathrm{CVaR}_\epsilon(\alpha(x)^\top \xi + \beta(x)) \leq 0 \right\}. \tag{3}$$

A tractable reformulation of (3) can be given based on [5, Corollary 5.1]. More specifically, one can utilize that

$$\mathrm{CVaR}_\epsilon(\alpha(x)^\top \xi + \beta(x)) = \inf_{\tau \in \mathbb{R}} \left\{ \tau + \frac{1}{\epsilon} \mathbb{E}^{\mathbb{Q}} \left[\left(\alpha(x)^\top \xi + \beta(x) - \tau \right)^+ \right] \right\}, \tag{4}$$

and similarly to the proof in [6, Theorem 21] formulate the supremum of the expectation. Provide the tractable reformulation of the supremum of expectation based on [5, Corollary 5.1]. Similarly, you may omit the constraints of the uncertainty set given in [5, Corollary 5.1], i.e. $C\xi \leq d$. Note that $x^+ = \max\{x, 0\}$ and $\mathbb{E}$ stands for the expectation. Additional information for the reformulation worst-case CVaR can be found in [5, Section 7.1].

4. Formulate the deterministic problem that the system operator needs to solve in real-time operation (RT) for each realization of wind power production, while having as fixed input the day-ahead decisions of conventional and wind power production units. This problem aims at minimizing the cost of re-dispatch actions. Since the constraints are formulated as “hard” constraints in this problem, i.e. no violation is allowed, the actions of load shedding and wind spilling need to be included to ensure feasibility of the problem. The action of load shedding is penalized with 1200\$/MWh and wind spillage is cost-free.

5. Now, model DR can be solved in order to obtain the first-stage variables (i.e., energy and reserve dispatch at the day-ahead stage). The value of ϵ is equal to 0.5 % and the ∞-norm is used in the tractable

reformulations of the objective function and chance constraints, hence the final programs to be solved are linear. Moreover, a training dataset $\widehat{\Xi}_N$ that contains N samples and a testing dataset $\widehat{\Psi}$ with 100 samples are introduced. The Wasserstein radius is defined by a vector $\mathcal{R} = [0\, 10^{[-4:0.2:-2]}]$. The following steps need to be followed:

(a) Solve DR model for $N = 10$ and $N = 100$ and for each value of ρ in vector $\mathcal{R}$ in order to obtain the values of first-stage variables. The day-ahead cost can now be calculated.

(b) Having the first-stage solutions as fixed input to RT model, solve RT model and calculate the real-time cost for each wind power realization in $\widehat{\Psi}$. The expected real-time cost can be calculated as the average for all realizations.

(c) For each N, calculate the total cost as the sum of the day-ahead and expected real-time cost for each value of radius in $\mathcal{R}$. Generate two plots that illustrate the total cost against the Wasserstein radius for $N = 10$ and $N = 100$. Report the minimum total cost obtained and the optimal radius that yields the minimum total cost for $N = 10$ and $N = 100$. Provide some comments regarding the results. Is there a relationship between the number of samples N used to approximate the empirical distribution and the minimum cost obtained? Do you notice any relation between the number of samples N and the optimal value of ρ?

The training and testing datasets are provided. The training datasets are named "Training_Dataset_N10.mat" and "Training_Dataset_N100.mat", while the testing dataset is "Testing_Dataset.mat". Please, implement the models using a proper solver, e.g., MOSEK, CPLEX or Gurobi, in your favorite software, e.g., Matlab (using YALMIP), Python, GAMS or Julia.

4 Lessons Learned - Reflection

Solving this assignment results in getting familiar with the formulation of distributionally robust problems in the context of power system operations. In no more than half a page, please list the 2-3 main points that you think you should remember for the next time you utilize distributionally robust optimization. How do you think the number of samples used to approximate the empirical distribution affect the solutions obtained? Moreover, one would note that varying the radius of the Wasserstein ball results in constructing increasingly conservative robust optimization problems. What type of problem is solved when the radius ρ is equal to zero and how would one pick the optimal radius ρ for the problem at hand to be solved? Additionally, different types of ambiguity sets can be used to define the family of distributions that we take into account in the optimization problems. Could you mention a couple of alternatives?

In case there were issues that came up with coding and making the simulations run correctly, please list these issues. What should you remember to do in some specific way, or avoid, next time you code such a program?

References

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