Perfect and Imperfect Competition in Electricity Markets

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1 Introduction

This project aims at analyzing the Nash equilibrium in electricity markets under different types of competition. The first part considers a perfectly competitive two-stage market (day-ahead and real-time) where participants take prices parametrically and cannot anticipate their influence on market price formation. In this part, the market-clearing problem writes as both a centralized *optimization* problem and an *equilibrium* problem. You can choose either a complementarity framework or a distributed optimization approach to tackle the equilibrium problem formulation. In the second part, a one-stage (day-ahead only) imperfect market is considered in view of Cournot duopoly model. This part aims to analyze the market power of generating companies and how they deal with asymmetry of information.

The objectives to be achieved at the end of the assignment are the following:

- Learn the concepts of perfect and imperfect competition in electricity markets.
- Formulate stochastic market clearing as centralized optimization and equilibrium problems.

- Formulate and solve a mixed-complementarity problem (MCP) and its distributed counterpart via alternating direction method of multipliers (ADMM).
- Analyze the market power of power producers via Cournot duopoly model.
- Learn and investigate the effects of information asymmetry using the Bayesian game.

2 Perfectly competitive market equilibrium

In this part, a two-stage electricity market under uncertainty is solved as an equilibrium problem. As opposed to the second part, all market participants are assumed to be price-takers. You will consider a stochastic market clearing as a centralized optimization problem and as its equilibrium counterpart. The market outcomes yielded by two approaches coincide, but they represent different visions on the market organization and involve distinct modeling attributes.

The models are applied to the test system consisting of three conventional generators, one wind power farm and a single inelastic load. The parameters of conventional units are collected in Table 1. The installed capacity of the wind farm amounts to 70MW, and its real-time output is described by a probabilistic forecast listed in Table 2. The load amounts to 200MW and can be curtailed at the real-time stage at expense of 100$/MWh. The intertemporal and unit commitment constraints are not envisaged in this project.

Table 1: Parameters of conventional generating units

Unit	Power capacity [MW]	Reserve capacity [MW]	Marginal cost [$/MWh]
G_1	100	50	20
G_2	90	15	15
G_3	80	0	10

This part involves the following steps:

Table 2: Probabilistic forecast of wind power output

Scenario	ω_1	ω_2	ω_3	ω_4	ω_5
Probability	0.15	0.2	0.3	0.2	0.15
Output [MW]	70	50	40	35	20

S1. Formulate the stochastic market-clearing problem as an optimization problem of the following structure:

$$\min_{x,y_\omega} \quad c^{\text{DA}}(x) + \mathbb{E}_\omega\{c^{\text{RT}}(y_\omega)\} \tag{1a}$$
$$\text{s.t.} \quad g^{\text{DA}}(x) \leq 0 \tag{1b}$$
$$g^{\text{RT}}(y_\omega) \leq 0 \; \forall\omega \tag{1c}$$
$$g^{\text{DA-RT}}(x, y_\omega) \leq 0 \; \forall\omega \tag{1d}$$
$$h^{\text{DA}}(x) = 0 : \lambda^{\text{DA}} \tag{1e}$$
$$h^{\text{RT}}(x, y_\omega) = 0 : \lambda^{\text{RT}}_\omega \tag{1f}$$

where x is a vector of the first-stage (day-ahead) dispatch decision variables and y_ω is a vector of the second-stage (real-time) adjustment decisions. The objective function (1a) consists of the day-ahead dispatch cost and the expectation of the real-time cost. A set of inequality constraints (1b)-(1d) defines the day-ahead and real-time dispatch limits as well as coupling constraints. Equality constraints (1e) and (1f) are power balance constraints that provide the dual variables defining the prices at the day-ahead and real-time stages. Please, find additional information on the centralized problem formulation in [1, Section II-A]. This centralized market-clearing setup ends up in a linear programming problem. Implement the model using a proper solver, e.g., MOSEK, CPLEX or Gurobi, in your favorite software, e.g., GAMS, Python, or Julia, [1].

In the following choose between S2 and S3.

S2. Using a mixed-complementarity programming (MCP), formulate the stochastic market clearing as an equilibrium problem. To do so, first define profit maximization problems for each market participant in the following form:

[1] If you choose task S2, be aware that the required solver for this task is only available for GAMS or Julia.

$$\left.\begin{cases} \max\limits_{x_i, y_{i\omega}} & \pi_i(x_i, y_{i\omega}) := \pi_i^{\text{DA}}(x_i) + \mathbb{E}_\omega\{\pi_i^{\text{RT}}(y_{i\omega})\} \\ \text{s.t.} & g^{\text{DA}}(x_i) \leq 0 \\ & g^{\text{RT}}(y_{i\omega}) \leq 0 \ \forall\omega \\ & g^{\text{DA-RT}}(x_i, y_{i\omega}) \leq 0 \ \forall\omega \end{cases}\right\} \forall \text{ generating unit } i, \tag{2}$$

$$\left.\begin{cases} \max\limits_{y_{l\omega}} & \pi_l(y_{l\omega}) := \mathbb{E}_\omega\{\pi_l^{\text{RT}}(y_{l\omega})\} \\ \text{s.t.} & g^{\text{RT}}(y_{l\omega}) \leq 0 \ \forall\omega \end{cases}\right\} \forall \text{ load } l, \tag{3}$$

where π is a pay-off function of a market participant that is computed as a product of dispatch quantity and difference of market price and production cost, e.g. the day-ahead profit of generator i writes as $\pi_i^{\text{DA}} = x_i(\lambda^{\text{DA}} - c_i)$. For inelastic loads, π_l^{RT} is defined as a minus curtailment cost, e.g., $\pi_l^{\text{RT}} = y_{l\omega}(\lambda_\omega^{\text{RT}} - 100)$. Additional information on these formulations can be found in [2]: refer to (1a) in [2] in case of conventional generating units; refer to (1c) in [2] in case of wind power units. In case of inelastic loads, the formulation is similar to [3, (14)-(15)].

Once formulations (2) and (3) are obtained, derive the corresponding Karush-Kuhn-Tucker (KKT) conditions in a form of [3, Appendix A]. How do they differ from those of problem (1)? Encapsulate the KKTs by power balance conditions (1e) and (1f) and formulate the mixed-complementarity problem. Solve the problem using PATH solver either in GAMS [4] or Julia [5].

S3. Solve the stochastic market clearing in a distributed fashion using the exchange alternating direction method of multipliers (ADMM) [6, 7.3.2 Optimal Exchange]. The focus of this task is to design an iterative algorithm that brings each profit-maximization problem (2) and (3) to an equilibrium state where conditions (1e) and (1f) are fulfilled.

In this line, objective functions of problems (2) and (3) need to be augmented by (1e) and (1f) using the dual variables of these conditions, as explained by Algorithm 1. Here, ν is an iteration counter limited from above by $\nu^{\max}$, N^{DA} is a number of day-ahead market participants (four in this study), N^{RT} is a number of the real-time market participants (five in this study), ρ is a regularization factor, D is the system load at the day-ahead stage, $\overline{x}_k^{(\nu)}$ and $\overline{y}_{k\omega}^{(\nu)}$ are the average first and second-stage decisions at iteration ν. Notice, for the loads, the primal update does not include the day-ahead stage.

The algorithm terminates when the regularization terms at both day-ahead and real-time stages are zero, meaning that the equilibrium conditions (1e) and (1f) are met. Though each profit-maximization problem is convex, there might occur some convergence issues, i.e., residuals oscillation, cycling, etc. In case it happens, employ the adaptive regularization factor ρ as explained in [6, 3.4.1 Varying Penalty Parameter].

Algorithm 1: Sharing ADMM for stochastic market clearing

Data: $\nu^{\max}, \lambda^{\mathrm{DA}(0)}, \lambda_\omega^{\mathrm{RT}(0)}, r^{\mathrm{DA}(0)}, r_\omega^{\mathrm{RT}(0)}, \rho$

for *ν from 0 to ν^{max}* **do**

Primal updates for each market participant k:

$$(x_k^{(\nu+1)}, y_{k\omega}^{(\nu+1)}) := \underset{x_k, y_{k\omega}}{\operatorname{argmax}} \Bigg\{ \pi_k(x_k, y_{k\omega}) - \lambda^{\mathrm{DA}(\nu)} x_k - \sum_\omega \lambda_\omega^{\mathrm{RT}(\nu)} y_{k\omega} - \frac{\rho}{2} \left\| x_k - r^{\mathrm{DA}(\nu)} \right\|^2 - \sum_\omega \frac{\rho}{2} \left\| y_{k\omega} - r_\omega^{\mathrm{RT}(\nu)} \right\|^2 \Bigg\}$$

Dual updates:

$$\lambda^{\mathrm{DA}(\nu+1)} := \lambda^{\mathrm{DA}(\nu)} - \rho(D - \textstyle\sum_k x_k^{(\nu)})$$

$$\lambda_\omega^{\mathrm{RT}(\nu+1)} := \lambda_\omega^{\mathrm{RT}(\nu)} - \rho \textstyle\sum_k y_{k_\omega}^{(\nu)} \ \forall \omega$$

Residual updates:

$$r^{\mathrm{DA}(\nu+1)} := x_k^{(\nu)} - N^{\mathrm{DA}} \overline{x}_k^{(\nu)} + D$$

$$r_\omega^{\mathrm{RT}(\nu+1)} := y_{k\omega}^{(\nu)} - N^{\mathrm{RT}} \overline{y}_{k\omega}^{(\nu)} \ \forall \omega$$

end

3 Imperfectly competitive market equilibrium

This section considers the imperfect market equilibrium where power producers are capable of influencing market prices depending on the submitted energy quantities (Cournot model). For simplicity, there are only two generators that need to decide on the amount of power to produce denoted by $x^i \in \mathbb{R}_+$. They each have a production marginal cost of $c^i \in \mathbb{R}_{++}$. The market price is a linearly decreasing function of the total production

$p(x^1; x^2) = a - b(x^1 + x^2)$, where a and b are positive constants. Each producer knows its marginal cost but not that of the other producer. It also knows that generators can be of two types: renewable (wind, solar, hydro) with a low marginal cost of c_l and coal-based with a high marginal cost of c_h. Knowing the total fraction of renewables to coal-based power plants in the country, each player estimates the probability of the other producer being renewable by $d \in (0; 1)$. This part involves the following steps:

S1. Suppose that producer 1 is coal-based and producer 2 is renewable and that both players know this information. Calculate the Nash equilibrium strategies.

S2. Incorporate producers' believe about a competitor being renewable. Formulate the corresponding Bayesian game and give a closed-form characterization of the Bayesian Nash equilibrium strategy. How do the optimal strategies change in d?

S3. Let $d = 1/3$, $a = 10$, $b = 3$, $c_l = 0.1$, $c_h = 1$. Evaluate the Bayesian Nash equilibrium strategy for each player and compare that to the full information Nash equilibrium.

4 Lessons Learned – Reflection

In this assignment, you were modeling perfectly and imperfectly competitive electricity markets. Please, discuss the main differences between the two types of competition shortly. As a regulator, why would you enhance the competition in the industry and what actions would you take?

Regarding the first part of the assignment, what are the advantages and disadvantages of the chosen equilibrium formulation in comparison with the centralized planning? In case of mixed-complementarity formulation, what extensions are possible to include in the MCP formulation that is impossible to account for in the centralized optimization (think concerning the diversity of market participant objectives)? In case of distributed ADMM implementation, discuss the differences regarding the security of private information (what information each player receives and what information it shares with other players) and scalability. Was there any computational challenge while implementing the ADMM algorithm?

Regarding the second part of the assignment, how does the market power of electricity producers affect pricing (hint: relate $p(x^1; x^2)$ and c^i)? How is it changed when producers face asymmetry of information? How do they value the knowledge about rival production technology?

References

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