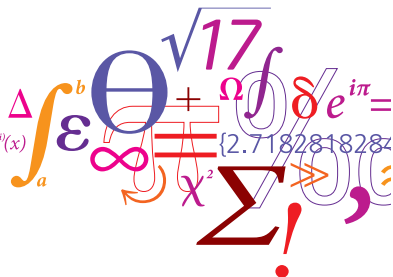


Optimal Power Flow (DC-OPF and AC-OPF)

DTU Summer School 2017

Spyros Chatzivasileiadis

$$f(x+\Delta x) = \sum_{i=0}^{\infty} \frac{(\Delta x)^i}{i!} f^{(i)}(x)$$




What is optimal power flow?



Optimal Power Flow (OPF)

- In its most realistic form, the OPF is a **non-linear, non-convex** problem, which includes both binary and continuous variables.
- Goal: minimize an objective function, respecting all physical, operational and technical constraints, such as:
 - Ohm's law and Kirchhoff laws
 - Operational limits of generators
 - Loading limits of transmission lines
 - Voltage levels
 - and many, many others
- **Disclaimer:**
 - Realistic OPF implementations include thousands of variables and constraints
 - Here we focus on the most "fundamental" formulations of OPF
 - These can be extended with several additional constraints to accurately model the problem and/or system at hand



Use of OPF in the industry

- RTE, France: they invented the OPF! (Carpentier, 1962)
 - Their focus is mostly on the optimization of the system operation and not so much on markets
 - They are working on the application of convex relaxations of OPF on their system (see Dan Molzahn's talk!)
- CAISO, California, USA
 - Electricity markets: An OPF runs every day (Day-Ahead), every hour, every 15 minutes, and every 5 minutes.
 - Depending on the problem they run a DC-OPF with unit commitment, a standard DC-OPF, or just Economic Dispatch

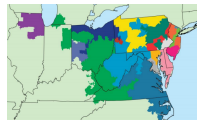


Use of OPF in the industry

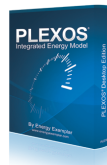
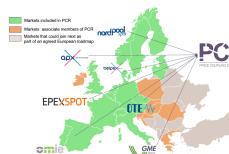
- PJM, East Coast, USA
 - Security-Constrained Economic Dispatch every 5 minutes
 - final test phase of optimal voltage control every 5 minutes (they use AC-OPF as well)

- EUPHEMIA
 - one common algorithm to calculate electricity prices across Europe, and allocate cross border capacity on a day-ahead basis
 - 19 European countries, over 150 million EUR in matched trades daily

- PLEXOS
 - June 2000: PLEXOS was first-to-market with electric power market simulation based entirely on mathematical programming
 - Features: generation capacity expansion planning, transmission expansion planning, hydro-thermal coordination, ancillary services



Towards the Single European Market: Next Steps



Summarizing

- DC-OPF
 - market clearing uses DC-OPF (at the moment)
 - convex
 - can solve fast; can be applied in very large problems
- AC-OPF
 - primarily used for optimization of operation and control actions
 - non-convex (in its original form)
 - continuous efforts to decrease computation time and increase system size
- Trends
 - Incorporating uncertainty
 - Decomposition (and other) methods to solve very large problems
 - Guarantees for a global minimum (convex relaxations)
 - Market design
 - Coupled energy networks and markets, e.g. gas and electricity



Outline

- Economic Dispatch
- DC-OPF
- AC-OPF



Outline

- Economic Dispatch
 - used in power exchanges, e.g. EPEX, etc.
 - Supply must meet demand
 - Generator limits



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 - extends Economic Dispatch
 - considers the power flows! (in a linearized form)
 - includes the power flow limits of the lines
 - only active power; no losses



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- Economic Dispatch
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- DC-OPF
 - extends Economic Dispatch
 - considers the power flows! (in a linearized form)
 - includes the power flow limits of the lines
 - only active power; no losses
- AC-OPF
 - full AC power flow equations
 - active and reactive power flow
 - current, voltage
 - losses



Economic Dispatch

$$\min \sum_i c_i P_{G_i}$$

subject to:

$$P_{G_i}^{min} \leq P_{G_i} \leq P_{G_i}^{max}$$

$$\sum_i P_{G_i} = P_D$$



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- The Economic Dispatch does not consider any network flows or network constraints!



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- The Economic Dispatch does not consider any network flows or network constraints!
- We assume a copperplate network, i.e. a **lossless and unrestricted flow** of electricity from A to B.



Can we solve the economic dispatch problem without using an optimization solver?

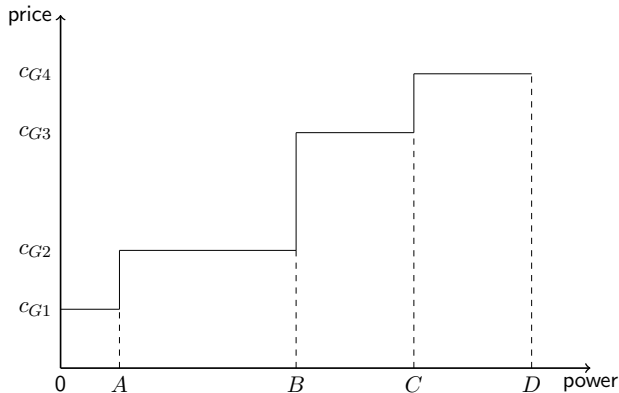


Can we solve the economic dispatch problem without using an optimization solver?

Yes! With the help of the merit order curve.



The Merit-Order Curve



$$A = P_{G1}^{max}$$

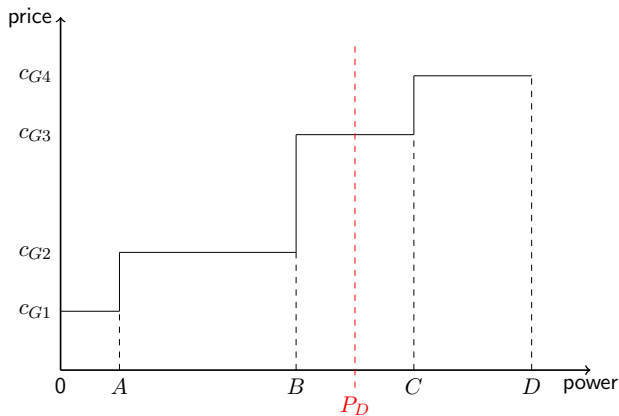
$$B = A + P_{G2}^{max}$$

$$C = B + P_{G3}^{max}$$

$$D = C + P_{G4}^{max}$$



The Merit-Order Curve



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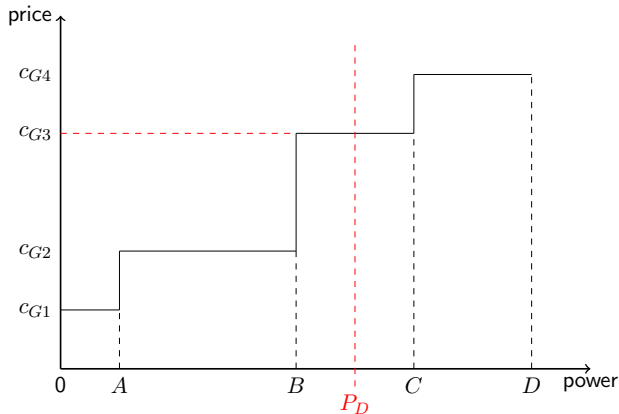
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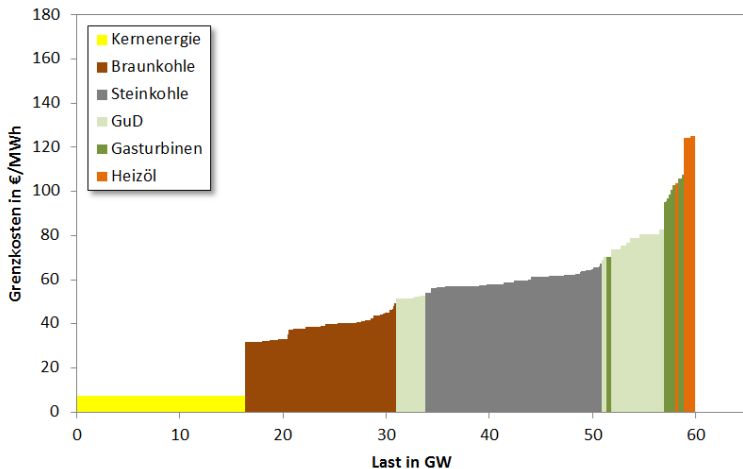
The Merit-Order Curve



- c_{G3} is the system marginal price
- Gens G1 and G2 are fully dispatched
- Gen G4 is not dispatched at all
- Gen G3 is partially dispatched
 - G3 is the “marginal generator”

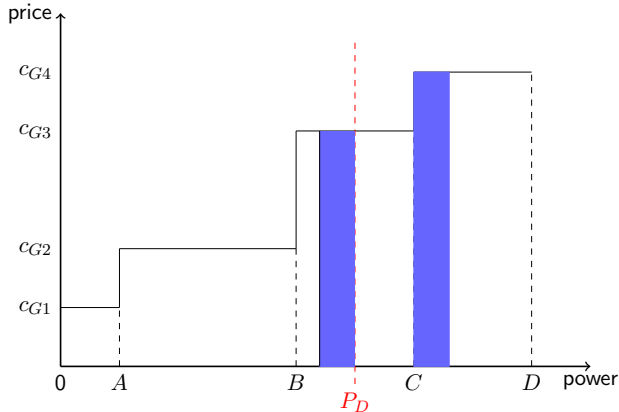


The Merit-Order Curve: An Example



Merit-Order of the German conventional generation in 2008. Source: Forschungsstelle für Energiewirtschaft e. V.

Merit-Order Curve, Marginal Generators, and Line Congestion



- Although G3 has enough capacity, it cannot produce enough to cover the demand due to line congestion
- Instead G4, a more expensive gen that does not contribute to the line congestion, must produce the missing power

- In a DC-OPF context, there is no longer a single system marginal price (we will observe different nodal prices in different nodes)

DC-OPF

$$\min \sum_i c_i P_{G_i}$$

subject to:

$$P_{G_i}^{min} \leq P_{G_i} \leq P_{G_i}^{max}$$

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subject to:

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$$\mathbf{B} \cdot \boldsymbol{\theta} = \mathbf{P}_G - \mathbf{P}_D$$

$$\frac{1}{x_{ij}}(\theta_i - \theta_j) \leq P_{ij,max}$$



DC-OPF

$$\min \sum_i c_i P_{G_i}$$

subject to:

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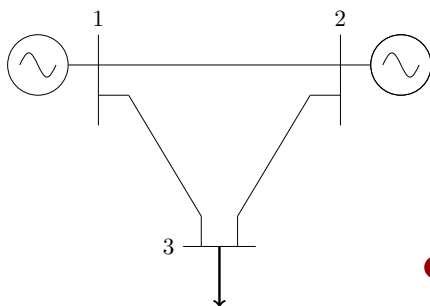
$$\mathbf{B} \cdot \boldsymbol{\theta} = \mathbf{P}_G - \mathbf{P}_D$$

$$\frac{1}{x_{ij}}(\theta_i - \theta_j) \leq P_{ij,max}$$

- The DC-OPF with the standard power flow equations contains both the power generation \mathbf{P}_G and the voltage angles $\boldsymbol{\theta}$ in the vector of the optimization variables.



Exercise



$$c_{G1} = 60 \text{ \$/MWh}, c_{G2} = 120 \text{ \$/MWh}$$

$$P_{load} = 150 \text{ MW}$$

$$P_{G1}^{max} = 100 \text{ MW}, P_{G2}^{max} = 200 \text{ MW}$$

$$X_{12} = 0.1 \text{ pu}, X_{13} = 0.3 \text{ pu}, X_{23} = 0.1 \text{ pu}, \text{BaseMVA} = 100 \text{ MVA}$$

$$P_{13}^{max} = 40 \text{ MW (line limit)}$$

- ① What are the optimization variables? For the optimization vector
- ② Formulate the objective function
- ③ Formulate the constraints



DC-OPF in Matlab

linprog

Solve linear programming problems

Linear programming solver

Finds the minimum of a problem specified by

$$\min_x f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq, \\ lb \leq x \leq ub. \end{cases}$$

f , x , b , beq , lb , and ub are vectors, and A and Aeq are matrices.

Syntax

```
x = linprog(f,A,b)
x = linprog(f,A,b,Aeq,beq)
x = linprog(f,A,b,Aeq,beq,lb,ub)
x = linprog(f,A,b,Aeq,beq,lb,ub,x0)
x = linprog(f,A,b,Aeq,beq,lb,ub,x0,options)
x = linprog(problem)
[x,fval] = linprog( __ )
[x,fval,exitflag,output] = linprog( __ )
[x,fval,exitflag,output,lambda] = linprog( __ )
```

How would you transfer your
problem formulation to Matlab?



Discussion Points

- $\sin \delta \approx \delta$
 - δ is in **rad**!

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 - θ is in rad, \Rightarrow dimensionless
 - **P must be in p.u.**



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- Bus Admittance Matrix \mathbf{B} in **DC-OPF**
 - $b_{ij} = \frac{1}{x_{ij}} \Rightarrow$ positive
 - all **off-diagonal** elements are **non-positive** (zero or negative)
 - all **diagonal** elements are **positive**
 - AC-OPF: This differs from the case where $z_{ij} = r_{ij} + jx_{ij}$. In that case, it is $y_{ij} = g_{ij} + jb_{ij}$ with b_{ij} is negative.



Discussion Points

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- If the DC-OPF does not converge, check that the admittance matrix \mathbf{B} is correct!



4-slide “break”

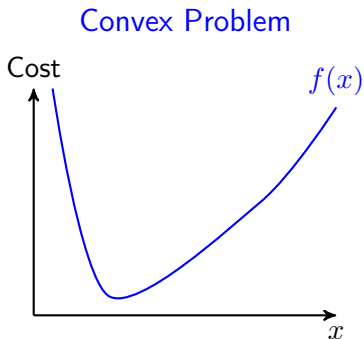
DC-OPF: linear program = convex

AC-OPF: non-linear non-convex problem in its original form
⇒ recent efforts to convexify the problem

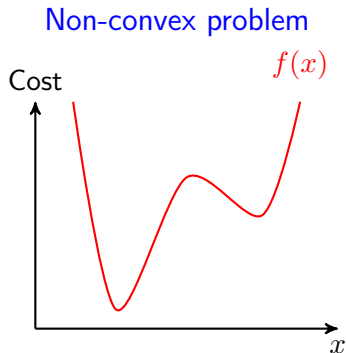
Why?



Convex vs. Non-convex Problem



One global minimum



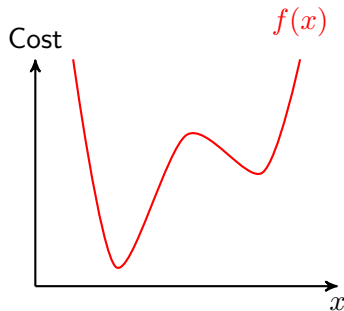
Several local minima



Several local minima: So what?

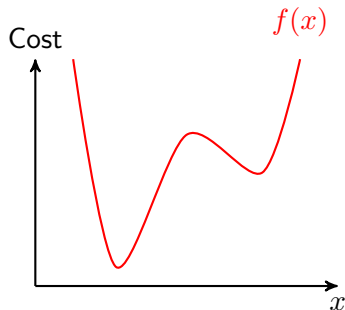
Example: Optimal Power Flow Problem

- Assume that the difference in the cost function of a local minimum versus a global minimum is 5%
- The total electric energy cost in the US is ≈ 400 Billion\$/year
- 5% amounts to 20 billion US\$ in economic losses per year
- Even 1% difference is huge
- Convex problems guarantee that we find a global minimum \Rightarrow convexify the OPF problem



Convexifying the Optimal Power Flow problem (OPF)

- Convex relaxations transform the OPF to a convex Semi-Definite Program (SDP)



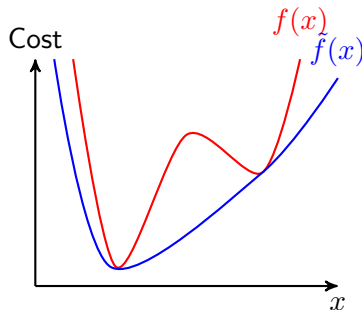
Convex Relaxation

¹Javad Lavaei and Steven H Low. "Zero duality gap in optimal power flow problem". In: *IEEE Transactions on Power Systems* 27.1 (2012), pp. 92–107



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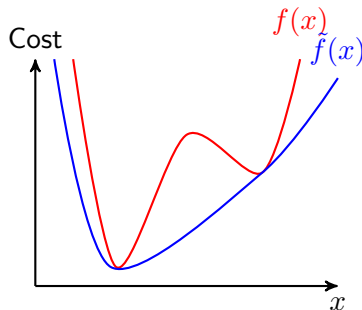
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Convexifying the Optimal Power Flow problem (OPF)

- Convex relaxations transform the OPF to a convex Semi-Definite Program (SDP)
- Under certain conditions, the obtained solution is the global optimum to the original OPF problem¹



Convex Relaxation

¹Javad Lavaei and Steven H Low. "Zero duality gap in optimal power flow problem". In: *IEEE Transactions on Power Systems* 27.1 (2012), pp. 92–107



Break is over... More in Dan Molzahn's lecture tomorrow!



AC-OPF

- Minimize

Costs, Line Losses, other?

- subject to:



AC-OPF

- Minimize

Costs, Line Losses, other?

- subject to:

AC Power Flow equations

Line Flow Constraints

Generator Active Power Limits

Generator Reactive Power Limits

Voltage Magnitude Limits

(Voltage Angle limits to improve solvability)

(maybe other equipment constraints)



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Line Current Limits

Apparent Power Flow limits

Active Power Flow limits

- Optimization vector: $[P \ Q \ V \ \theta]^T$



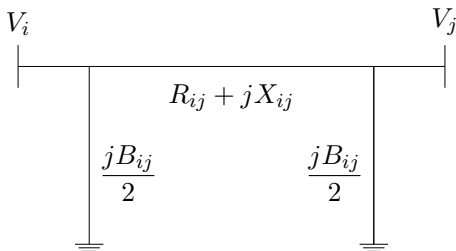
AC-OPF²

obj.function	$\min c^T P_G$
AC flow	$S_G - S_L = \text{diag}(\bar{V}) \bar{Y}_{\text{bus}}^* \bar{V}^*$
Line Current	$ \bar{Y}_{\text{line},i \rightarrow j} \bar{V} \leq I_{\text{line},\text{max}}$ $ \bar{Y}_{\text{line},j \rightarrow i} \bar{V} \leq I_{\text{line},\text{max}}$
or Apparent Flow	$ \bar{V}_i \bar{Y}_{\text{line},i \rightarrow j, \text{i-row}}^* \bar{V}^* \leq S_{i \rightarrow j, \text{max}}$ $ \bar{V}_j \bar{Y}_{\text{line},j \rightarrow i, \text{j-row}}^* \bar{V}^* \leq S_{j \rightarrow i, \text{max}}$
Gen. Active Power	$0 \leq P_G \leq P_{G, \text{max}}$
Gen. Reactive Power	$-Q_{G, \text{max}} \leq Q_G \leq Q_{G, \text{max}}$
Voltage Magnitude	$V_{\text{min}} \leq V \leq V_{\text{max}}$
Voltage Magnitude	$V_{\text{min}} \leq V \leq V_{\text{max}}$
Voltage Angle	$\theta_{\text{min}} \leq \theta \leq \theta_{\text{max}}$

²All shown variables are vectors or matrices. The bar above a variable denotes complex numbers. $(\cdot)^*$ denotes the complex conjugate. To simplify notation, the bar denoting a complex number is dropped in the following slides. **Attention! The current flow constraints are defined as vectors, i.e. for all lines. The apparent power line constraints are defined per line.**



Current flow along a line



π -model of the line

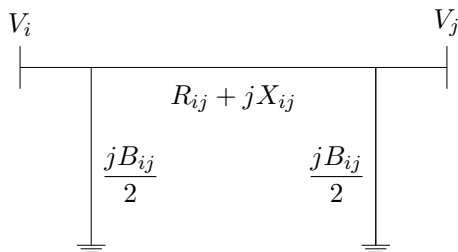
It is:

$$y_{ij} = \frac{1}{R_{ij} + jX_{ij}}$$

$$y_{sh,i} = j\frac{B_{ij}}{2} + \text{other shunt elements connected to that b}$$



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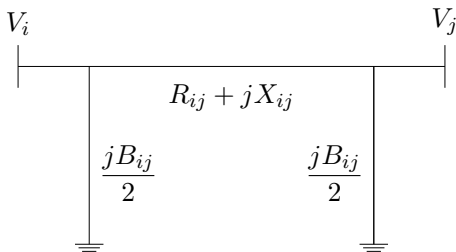
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$$y_{sh,i} = j\frac{B_{ij}}{2} + \text{other shunt elements connected to that b}$$

$$i \rightarrow j : I_{i \rightarrow j} = y_{sh,i}V_i + y_{ij}(V_i - V_j) \Rightarrow I_{i \rightarrow j} = \begin{bmatrix} y_{sh,i} + y_{ij} & -y_{ij} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix}$$



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Line Admittance Matrix Y_{line}

- Y_{line} is an $L \times N$ matrix, where L is the number of lines and N is the number of nodes
- if row k corresponds to line $i - j$:
 - $Y_{\text{line},ki} = y_{sh,i} + y_{ij}$
 - $Y_{\text{line},kj} = -y_{ij}$
- $y_{ij} = \frac{1}{R_{ij} + jX_{ij}}$ is the admittance of line ij
- $y_{sh,i}$ is the shunt capacitance $jB_{ij}/2$ of the π -model of the line
- We must create two Y_{line} matrices. One for $i \rightarrow j$ and one for $j \rightarrow i$



Bus Admittance Matrix Y_{bus}

$$S_i = V_i I_i^*$$

$$I_i = \sum_k I_{ik}, \text{ where } k \text{ are all the buses connected to bus } i$$

Example: Assume there is a line between nodes $i - m$, and $i - n$. It is

$$\begin{aligned} I_i &= I_{im} + I_{in} \\ &= (y_{sh,i}^{i \rightarrow m} + y_{im})V_i - y_{im}V_m + (y_{sh,i}^{i \rightarrow n} + y_{in})V_i - y_{in}V_n \\ &= (y_{sh,i}^{i \rightarrow m} + y_{im} + y_{sh,i}^{i \rightarrow n} + y_{in})V_i - y_{im}V_m - y_{in}V_n \end{aligned}$$

$$I_i = \underbrace{[y_{sh,im} + y_{im} + y_{sh,in} + y_{in}]}_{Y_{bus,ii}} \underbrace{[-y_{im}]}_{Y_{bus,im}} \underbrace{[-y_{in}]}_{Y_{bus,in}} [V_i \ V_m \ V_n]^T$$



Bus Admittance Matrix Y_{bus}

- Y_{bus} is an $N \times N$ matrix, where N is the number of nodes
- diagonal elements: $Y_{\text{bus},ii} = y_{sh,i} + \sum_k y_{ik}$, where k are all the buses connected to bus i
- off-diagonal elements:
 - $Y_{\text{bus},ij} = -y_{ij}$ if nodes i and j are connected by a line³
 - $Y_{\text{bus},ij} = 0$ if nodes i and j are not connected
- $y_{ij} = \frac{1}{R_{ij} + jX_{ij}}$ is the admittance of line ij
- $y_{sh,i}$ are all shunt elements connected to bus i , including the shunt capacitance of the π -model of the line

³If there are more than one lines connecting the same nodes, then they must all be added to

$Y_{\text{bus},ij}, Y_{\text{bus},ii}, Y_{\text{bus},jj}$.



AC Power Flow Equations

$$\begin{aligned} S_i &= V_i I_i^* \\ &= V_i Y_{\text{bus}}^* V^* \end{aligned}$$

For all buses $S = [S_1 \dots S_N]^T$:

$$S_{\text{gen}} - S_{\text{load}} = \text{diag}(V) Y_{\text{bus}}^* V^*$$



From AC to DC Power Flow Equations

- The power flow along a line is:

$$S_{ij} = V_i I_{ij}^* = V_i (y_{sh,i}^* V_i^* + y_{ij}^* (V_i^* - V_j^*))$$

- Assume a negligible shunt conductance: $g_{sh,ij} = 0 \Rightarrow y_{sh,i} = jb_{sh,i}$.
- Given that $R \ll X$ in transmission systems, for the DC power flow we assume that $z_{ij} = r_{ij} + jx_{ij} \approx jx_{ij}$. Then $y_{ij} = -j \frac{1}{x_{ij}}$.
- Assume: $V_i = V_i \angle 0$ and $V_j = V_j \angle \delta$, with $\delta = \theta_j - \theta_i$.

$$\begin{aligned} I_{ij}^* &= -jb_{sh,i} V_i + j \frac{1}{x_{ij}} (V_i - (V_j \cos \delta - jV_j \sin \delta)) \\ &= -jb_{sh,i} V_i + j \frac{1}{x_{ij}} V_i - j \frac{1}{x_{ij}} V_j \cos \delta - \frac{1}{x_{ij}} V_j \sin \delta \end{aligned}$$



From AC to DC Power Flow Equations (cont.)

- Since V_i is a real number, it is:

$$P_{ij} = \Re\{S_{ij}\} = V_i \Re\{I_{ij}^*\} = -\frac{1}{x_{ij}} V_i V_j \sin \delta$$

- With $\delta = \theta_j - \theta_i$, it is:

$$P_{ij} = \frac{1}{x_{ij}} V_i V_j \sin(\theta_i - \theta_j)$$

- We further make the assumptions that:
 - V_i, V_j are constant and equal to 1 p.u.
 - $\sin \theta \approx \theta$, θ must be in rad

Then

$$P_{ij} = \frac{1}{x_{ij}} (\theta_i - \theta_j)$$



Some additional points...

- Nodal prices

In a market context, the nodal prices are:

- the **lagrangian multipliers** of the equality constraints $B\theta = P$
- of a **DC-OPF** (at the moment)
- with objective function the **minimization of costs**



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- Nodal prices

In a market context, the nodal prices are:

- the **lagrangian multipliers** of the equality constraints $B\theta = P$
- of a **DC-OPF** (at the moment)
- with objective function the **minimization of costs**

- Power Transfer Distribution Factors (PTDFs)

- PTDFs are linear sensitivities that relate the line flows to the power injections
- the DC-OPF can be formulated with respect to PTDFs
- PTDFs eliminate the need of θ as optimization variable
- In the zonal pricing in Europe PTDFs are used to model the flows between the zones



AC-OPF

- Resources about AC-OPF from the US Federal Energy Regulatory Commission (FERC)

`https://www.ferc.gov/industries/electric/indus-act/
market-planning/opf-papers.asp`

- Overview paper on Economic Dispatch and DC-OPF:

R.D. Christie, B. F. Wollenberg, I. Wangestein, Transmission Management in the Deregulated Environment, Proceedings of the IEEE, vol. 88, no. 2, February 2000



Wrap-up

DC-OPF

- market clearing uses DC-OPF (at the moment)
- convex
- can solve fast; can be applied in very large problems

but

- only active power flow
- no losses and no voltage levels

DC approximations more suitable for transmission systems

AC-OPF

- primarily used for optimization of operation and control actions
- full AC power flow equations

but

- non-convex (in its original form) no guarantee that we find the global optimum
- computationally expensive and possibly intractable for very large systems

efforts to decrease computation time and increase system size



Thank you!

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