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EES-UETP Course on Uncertainty in Electricity Markets and System Operation

Optimal reserve offer deliverability in co-optimized electricity markets: A two-stage robust optimization approach

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Contents

- Introduction
- Robust model
- Solution approach
- Numerical results
- Conclusions

Introduction

- Power system operation is exposed to uncertainty sources
  - Fluctuations in nodal net injections (demand, renewable-based generation)
  - Loss of system components
- Capability to withstand uncertainty realizations implemented via reserves

Reserves

- Different types of reserves according to practical operating manuals

- One general classification:

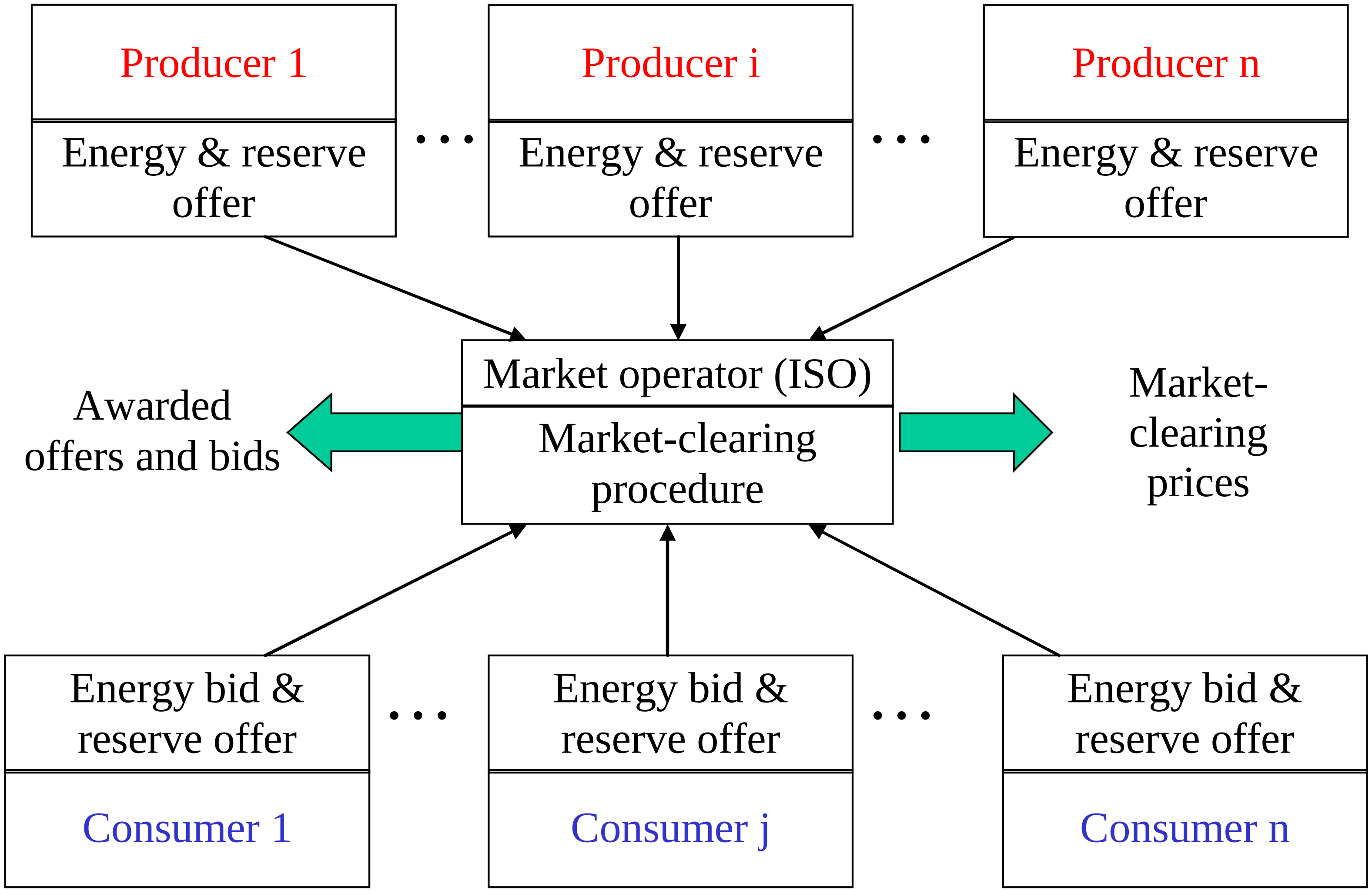
  - Synchronized or spinning reserves $\Rightarrow$ Provided by units that are scheduled on

  - Non-synchronized or non-spinning reserves $\Rightarrow$ Provided even by units that are scheduled off

Trading reserves

- Reserves are considered as tradable commodities in a similar way to energy
- Market participants are allowed to submit reserve offers
- Typical reserve offer $\Rightarrow$ Pair quantity-price
- Market operator is responsible for scheduling energy and reserves

Trading reserves



Trading reserves

- Current market implementations:
  - Sequential markets for energy and reserves (Europe) $\Rightarrow$ Simple albeit suboptimal
  - Co-optimized electricity markets (Greece, USA) $\Rightarrow$ Energy and reserves are jointly cleared

Goal of the co-optimization

- Determination of the awarded levels of energy and reserve offers that:
  - Maximize declared social welfare
  - Comply with operational limits
  - Guarantee power balance under the normal state and under any uncertainty realization

Problem definition

- Two different categories of system states:
  - Normal state $\Rightarrow$ Forecast uncertainty realizations
  - States under uncertainty
- Straightforward solution $\Rightarrow$ Replication of operational constraints (contingency-constrained model)

Contingency-constrained model

$Minimize\ c(x,r)$

Social welfare (energy and reserves)

$subject\ to$:

$g(x,r) \leq 0$

Pre-contingency constraints

$g_k(x,r,x_k) \leq 0 \quad \forall k \in \mathcal{C}$

Post-contingency constraints

- Large-scale mixed-integer program

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Contingency-constrained model

- Operational constraints (pre- and post-contingency):
    - Generation limits
    - Reserve limits
    - Network-related constraints $\Rightarrow$ dc load flow
- Mixed-integer linear programming

Contingency-constrained model

- Uncertainty realizations (nodal net injections and component outages) are characterized as contingencies indexed by k within the contingency set $\mathcal{C}$
  - $D_{bt}^{k} \Rightarrow$ Realizations of nodal net injections
  - A_{it}^{k}, A_{lt}^{k} $\Rightarrow$ 0/1 parameters modeling the unavailability/availability of generators and lines

Contingency-constrained model

- Contingency set $\mathcal{C}$ determined by:
  - Plausible realizations of nodal net injections

$$D_{bt}^{k} \in \mathcal{D}_{bt}$$

  - Security criterion (n – 1, n – 2, n – K, n – K^{G} – K^{L})

$$\boldsymbol{f}\left(\left\{A_{it}^{k}\right\}_{i\in I},\left\{A_{lt}^{k}\right\}_{l\in\mathcal{L}}\right) \le \mathbf{0}$$

Contingency-constrained model
Practical issues

- Problem dimension depends on the number of states

- Contingency set $\mathcal{C}$ is infinitely large

- Intractable model in practice

Practical solution

- Pre-specification of system reserve requirements based on engineering judgment

- Operation under uncertainty realization typically disregarded $\Rightarrow$ Use of forecast values

Practical solution for energy and reserve co-optimization

$$Minimize\ c(x,r)$$

$$subject\ to:$$

$$g(x,r) \leq 0$$

$$g^{req}(r) \leq 0$$

- Drawback $\Rightarrow$ Feasible deployment of reserve offers is not guaranteed even for sufficient reserve requirements!!

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Motivation for robust optimization

- Exact albeit intractable contingency-constrained model is essentially a worst-case problem

- Equivalent to a penalized contingency-constrained model

Motivation for robust optimization Equivalent penalized model

- Nodal power imbalance is allowed $\Rightarrow$ Slack variables
- Penalty term in the objective function associated with the worst-case system power imbalance
- Equivalence guaranteed by a sufficiently large penalty coefficient

Motivation for robust optimization
Equivalent penalized model

$$Minimize\ c(x,r) + C^I max_{k \in \mathcal{C}}(e^T \Delta P_k)$$

$subject\ to$:

$$g(x,r) \leq 0$$

$$g_k(x,r,x_k,\Delta P_k) \leq 0 \quad \forall k \in \mathcal{C}$$

- Also a large-scale mixed-integer program

Two-stage adaptive robust optimization

- Worst-case optimization setting $\Rightarrow$ Suitable for ensuring feasibility under all uncertainty realizations
- No accurate probabilistic information is required $\Rightarrow$ Suitable for renewable-based generation
- Problem size not dependent on the level of accuracy for uncertainty $\Rightarrow$ Convenient for tractability purposes

Two-stage adaptive robust optimization

- First stage:
  - Decisions that immunize against uncertainty realizations
- Second stage:
  - Worst-case uncertainty realizations and corresponding corrective actions

Uncertainty characterization

- Uncertainty realizations are represented by uncertainty-related decision variables

  - $d_{bt} \Rightarrow$ Nodal net injections

  - $a_{it}^{G}, a_{lt}^{L} \Rightarrow$ Loss of generators and lines

- Non-dependent on index k!!

Uncertainty characterization

- Feasibility space of uncertainty-related decision variables $\Rightarrow$ Pre-specified uncertainty set $\mathcal{U}$

$$a_{it}^{G}, a_{lt}^{L}, d_{bt} \in \mathcal{U}$$

Uncertainty set

- Upper and lower bounds for uncertainty-related decision variables $\Rightarrow$ Based on historical data or practical aspects

- Uncertainty budget $\Rightarrow$ Limit on the conservativeness of the solution based on engineering judgement

Uncertainty set: Nodal net injections

- Cardinality model:

$$d_{bt}^{f} - \Delta_{bt}^{dn} \leq d_{bt} \leq d_{bt}^{f} + \Delta_{bt}^{up}$$

$$\sum_{b \in \mathcal{B}^{u}} \left[\frac{max\{0, d_{bt} - d_{bt}^{f}\}}{\Delta_{bt}^{up}} + \frac{max\{0, d_{bt}^{f} - d_{bt}\}}{\Delta_{bt}^{dn}} \right] = K$$

- Tradeoff between accuracy and complexity

Uncertainty set: Nodal net injections

- Polyhedral uncertainty sets:

  - Worst-case uncertainty realization $\Rightarrow$ Extreme of the polytope

  - Equivalent discrete model based on binary variables

Uncertainty set: Nodal net injections

- Binary-variable-based cardinality model:

$$d_{bt} = d_{bt}^{f} + \Delta_{bt}^{up} a_{bt}^{up} - \Delta_{bt}^{dn} a_{bt}^{dn}$$

$$\sum_{b \in \mathcal{B}^u} \left(a_{bt}^{up} + a_{bt}^{dn} \right) = K$$

$$a_{bt}^{up}, a_{bt}^{dn} \in \{0,1\}$$

Uncertainty set: Component outages

- Straightforward use of 0/1 variables a_{it}^{G} and a_{lt}^{L}
- Uncertainty budget $\Rightarrow$ Security criterion
- Binary-variable-based cardinality model:

$$a_{it}^{G}, a_{lt}^{L} \in \{0,1\}$$

$$\boldsymbol{f}\left(\left\{a_{it}^{G}\right\}_{i\in I}, \left\{a_{lt}^{L}\right\}_{l\in \mathcal{L}}\right) \leq \mathbf{0}$$

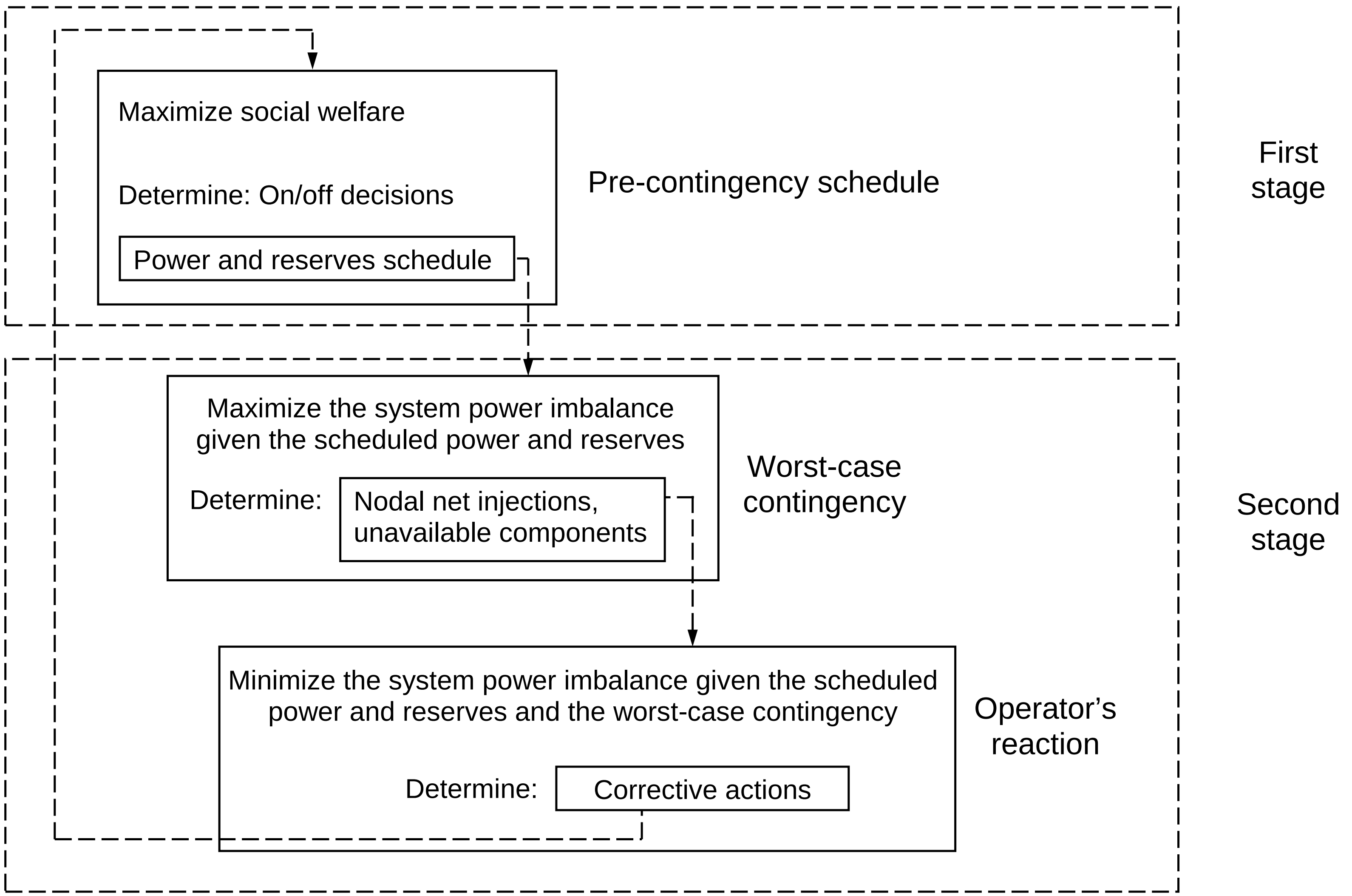
Robust counterpart

- Contingency constraints are replaced with an optimization problem to characterize the worst case

- Worst case $\Rightarrow$ Maximum damage (system power imbalance) associated with ALL contingencies implicitly modeled by a_{bt}^{up}, a_{bt}^{dn}, a_{it}^{G}, a_{lt}^{L},

- Robust counterpart $\Rightarrow$ Trilevel program

Trilevel robust counterpart



Trilevel robust counterpart

$$Minimize_{x,r}\ c(x,r) + C^I \Phi^{wc}(x,r)$$

subject to:

$$g(x,r) \leq 0$$

$$\Phi^{wc}(x,r) = max_a\ \delta(x,r,a)$$

subject to:

$$f^{dis}(a) \leq 0$$

$$\delta(x,r,a) = min_{x^u,\ \Delta P^u}\ (e^T \Delta P^u)$$

subject to:

$$g^u(x,r,a,x^u,\ \Delta P^u) \leq 0$$

Trilevel robust counterpart

- Two lowermost optimization problems replace post-contingency constraints $\Rightarrow$ Mixed-integer trilevel program

- Penalty term in the upper-level objective function is optimized in the two lowermost problems

- Two lowermost optimization problems $\Rightarrow$ Max-min optimization with a linear lower-level problem

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Suitable solution approaches

- Decomposition-based methods involving the iterative solution of a master problem and a max-min subproblem:
  - Benders decomposition
  - Column-and-constraint generation algorithm
- Optimal solution to the master problem and the max-min subproblem $\Rightarrow$ Finite convergence to global optimality

Column-and-constraint generation Master problem

- Relaxation of the trilevel counterpart

- Iteratively tighter by the addition of operating constraints built with information from the subproblem

Column-and-constraint generation Master problem

$$Minimize_{\alpha,x,r,\,x^{u,m},\Delta P^{u,m}} \quad c(x,r) + C^I \alpha$$

$$subject\ to:$$

$$g(x,r) \leq 0$$

$$\alpha \geq e^T \Delta P^{u,m};\ \forall m \in \mathcal{M}$$

$$g^u\left(x, r, a^{(m)}, x^{u,m}, \Delta P^{u,m}\right) \leq 0;\ \forall m \in \mathcal{M}$$

Column-and-constraint generation Max-min subproblem

- Two lowermost levels for given x and r from the master problem

$$
\begin{aligned}
&Maximize_a\ \delta(x, r, a)\\
&subject\ to:\\
&\qquad f^{dis}(a) \le 0\\
&\qquad \delta(x, r, a) = min_{x^u,\ \Delta P^u}\ (e^T \Delta P^u)\\
&\qquad\qquad subject\ to:\\
&\qquad\qquad g^u(x, r, a, x^u,\ \Delta P^u) \le 0
\end{aligned}
$$

Column-and-constraint generation Max-min subproblem

- Transformation to a single-level mixed-integer linear equivalent

  - Dual of the lower-level problem renders the original max-min a max-max $\Rightarrow$ Maximization problem (strong duality theorem)

  - Integer algebra results for the resulting bilinear terms

Dual of the lower-level problem

$$\delta(x, r, a) = max_{\pi}\, h^{u,dual}(\pi, x, r, a)$$

$$subject\ to:$$

$$g^{u,dual}(\pi, a) \leq 0$$

- At the optimum (strong duality theorem):

$$(e^T \Delta P^u) = h^{u,dual}(\pi, x, r, a)$$

Single-level equivalent for the subproblem

$$Maximize_{a,\pi}\, h^{u,dual}(\pi, x, r, a)$$

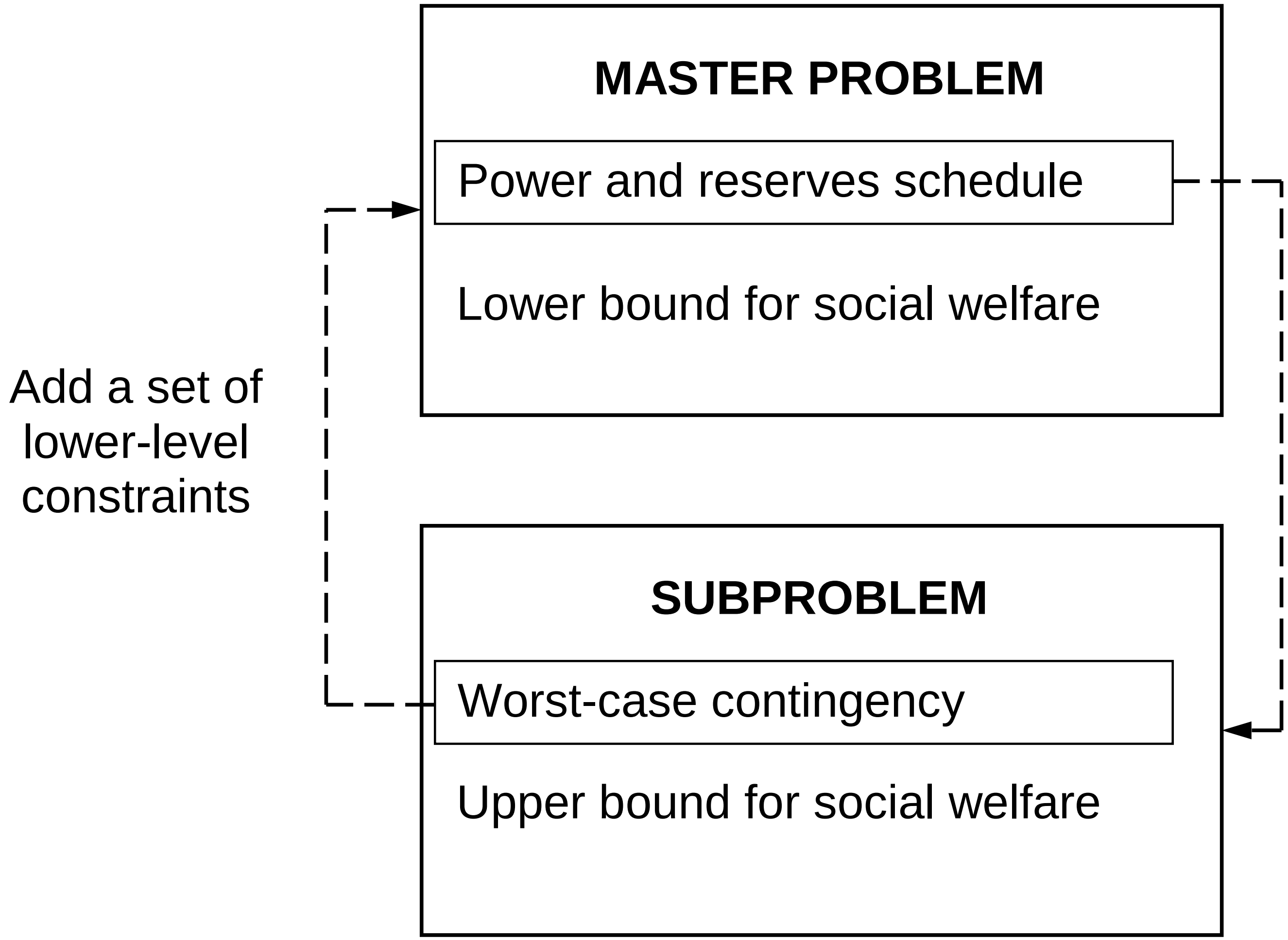
subject to:

$$f^{dis}(a) \leq 0$$

$$g^{u,dual}(\pi, a) \leq 0$$

- Bilinear terms in $h^{u,dual}(\pi, x, r, a)$ $\Rightarrow$ Well-known linearization scheme

Column-and-constraint generation



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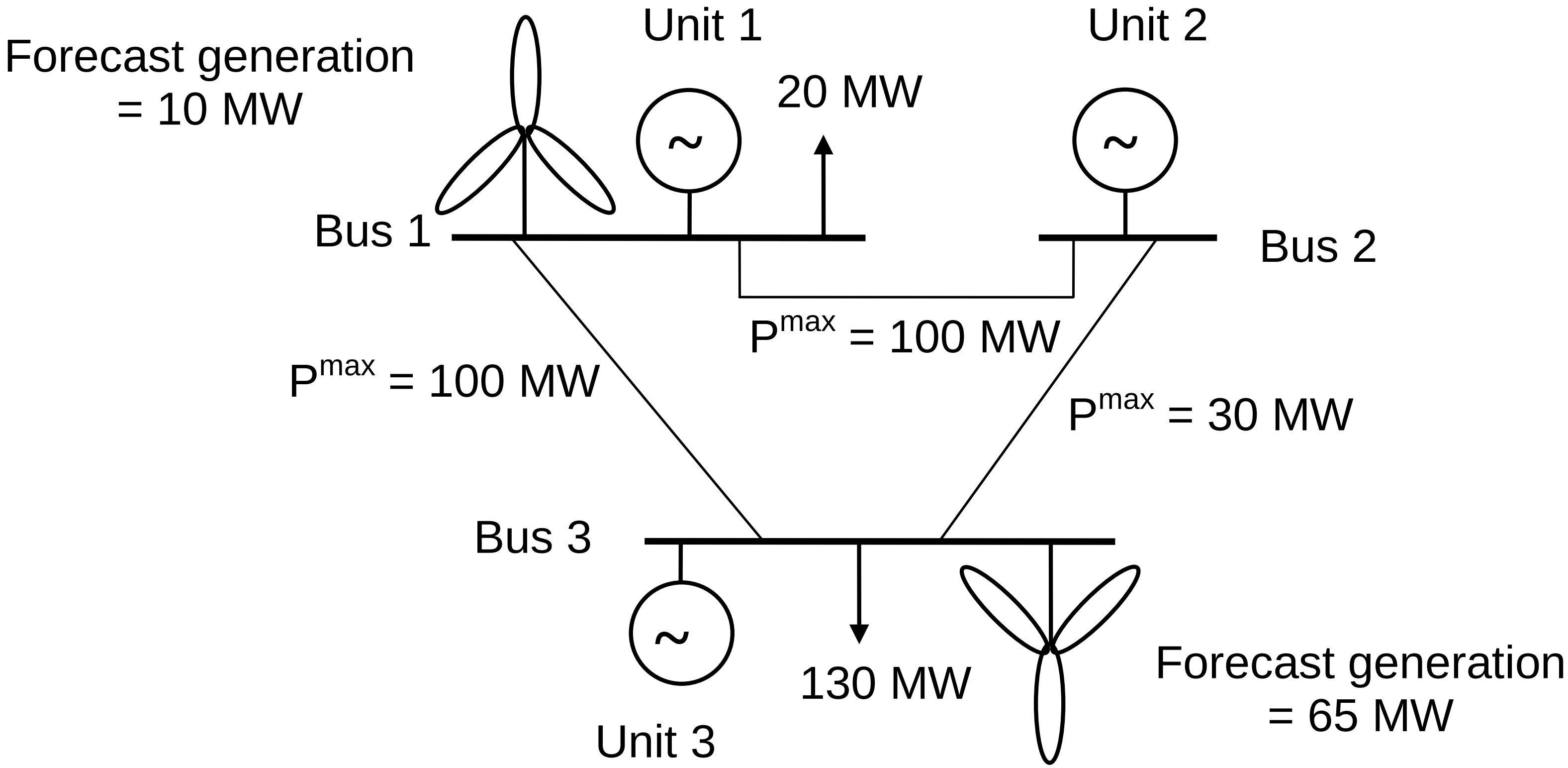
Case studies

- Illustrative three-bus, three-line, single-period example
- IEEE 118-bus system, 24 hourly periods
- CPLEX 12.6 under GAMS 24.2
- 4 Intel Xeon E7-4820 processors, 2.00 GHz, 768 GB of RAM

Illustrative single-period example



Illustrative single-period example

Unit	$\bar{P}, \bar{R}$ (MW)	$\underline{P}$ (MW)	C^v (\$/MWh)	C^f (\$)	C^{up} (\$/MW)	C^{dn} (\$/MW)
1	50	2.4	50.0	10.0	1.0	0.25
2	50	2.4	3.0	0.5	0.3	0.02
3	50	2.4	19.9	5.0	0.7	0.10

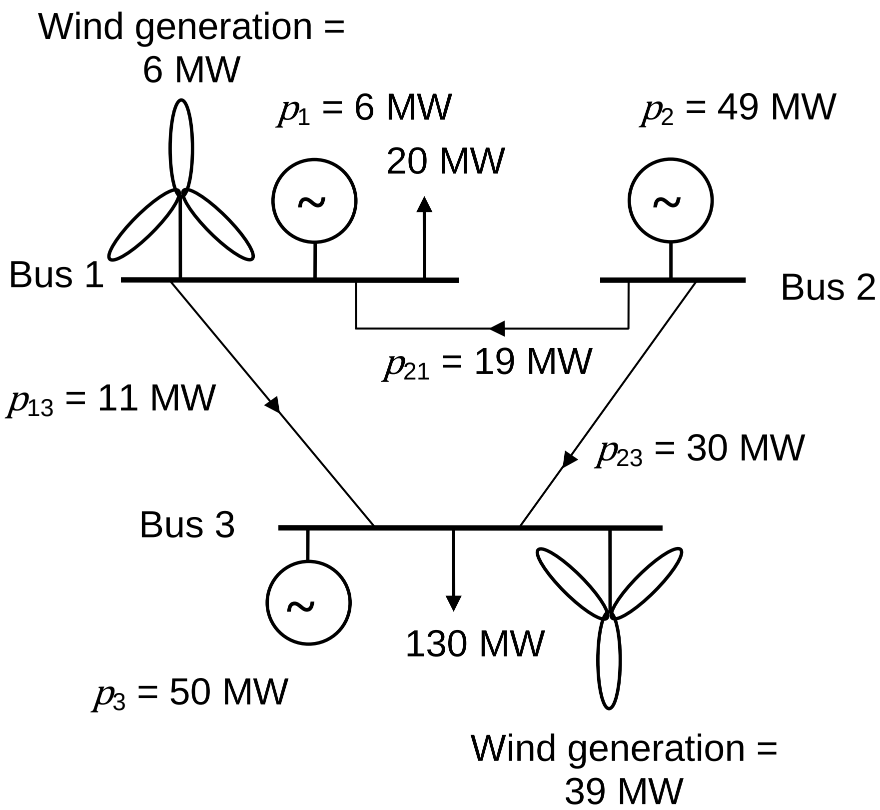
- All line reactances equal to 0.63 p.u.
- ±40% wind power fluctuation (±30 MW), no component outages

Illustrative single-period example Results

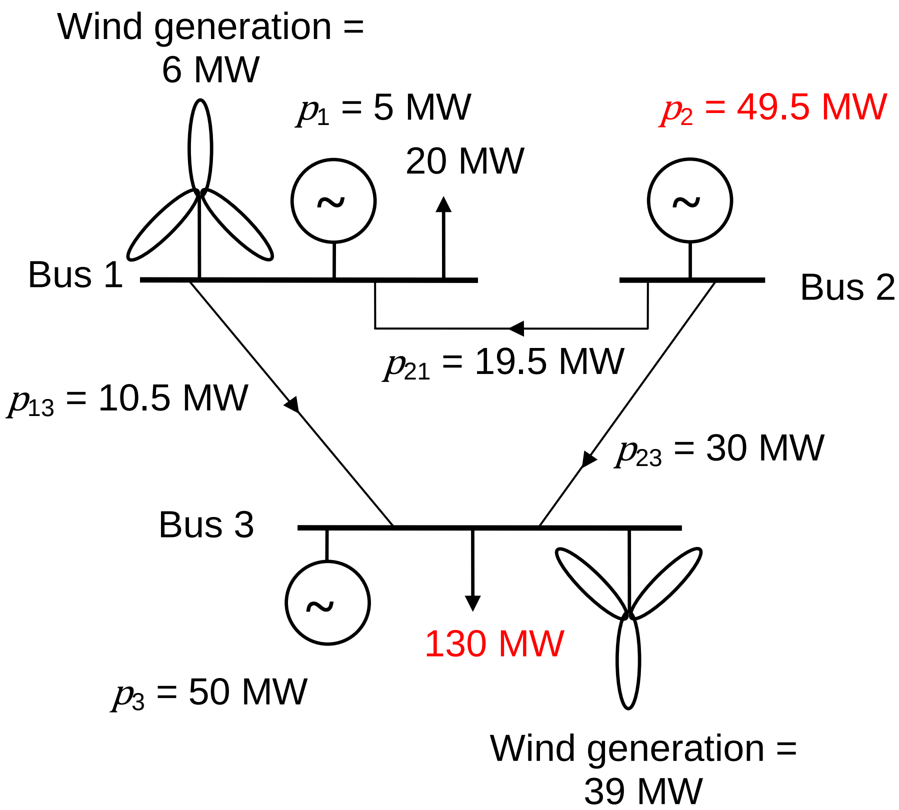
Unit	Under no uncertainty	Under uncertainty					
		Robust model			Conventional model		
	p_i (MW)	p_i (MW)	r_i^{up} (MW)	r_i^{dn} (MW)	p_i (MW)	r_i^{up} (MW)	r_i^{dn} (MW)
1	0	2.4	3.6	0	2.4	2.6	0
2	50	48.8	0.2	30	48.8	1.2	30
3	25	23.8	26.2	0	23.8	26.2	0
Cost ($)	653.0	778.1			777.4		

- Convergence in 3 iterations requiring less than 1 second

Illustrative single-period example
Worst-case operation



Robust solution

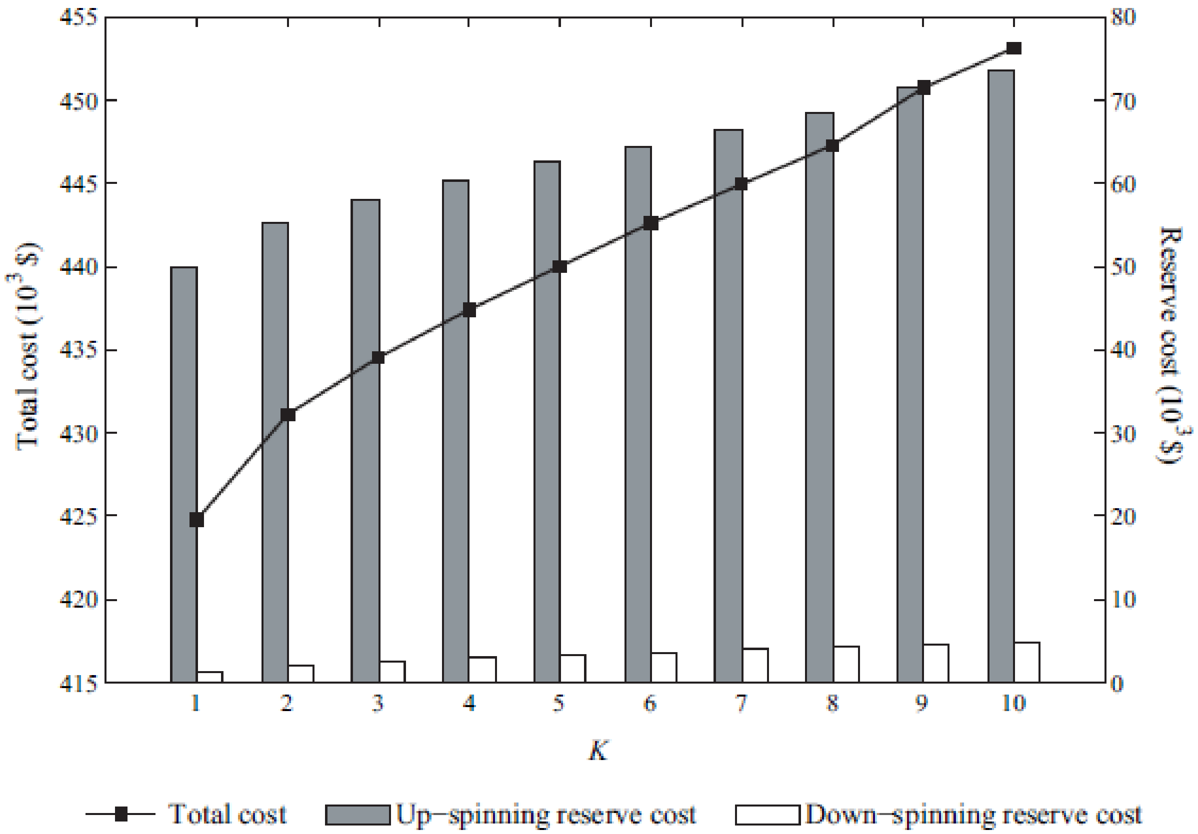


Conventional solution

IEEE 118-bus system

- 118 buses, 186 lines, 54 dispatchable units, 10 wind farms (25.8% wind penetration), 24 periods
- ±20% wind power fluctuation
- 66 contingencies (all dispatchable generators + 12 critical lines)
- 0.01-MW stopping criterion

IEEE 118-bus system Results



- Convergence in 8 iterations, 2.2 min/iteration

IEEE 118-bus system
Infeasibility of reserve requirements

K	Worst power Imbalance (MW)
1	1980.3
2	2675.5
3	3049.5
4	3479.7
5	3908.8
6	4328.4
7	4718.0
8	5107.6
9	5492.2
10	5863.4

- Feasibility $\Rightarrow$ 0.1%-6.8% cost increase

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Conclusions

- Robust optimization is suitable for co-optimized electricity markets under uncertainty

- Reserve deliverability is achieved with moderate reduction of social welfare

- Optimality is attained in a few iterations

Further research

- Incorporation of correlated uncertainty sources $\Rightarrow$ Alternative uncertainty sets

- Consideration of more sophisticated operational models (ac load flow, line switching)

- Analysis of alternative solution approaches to avoid the dual-based transformation

End of the presentation

Thanks for your attention!

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